General Analysis of a Point Dislocation

1. Point-dislocation force equivalent
2. Moment magnitude M_w
3. The seismic moment tensor

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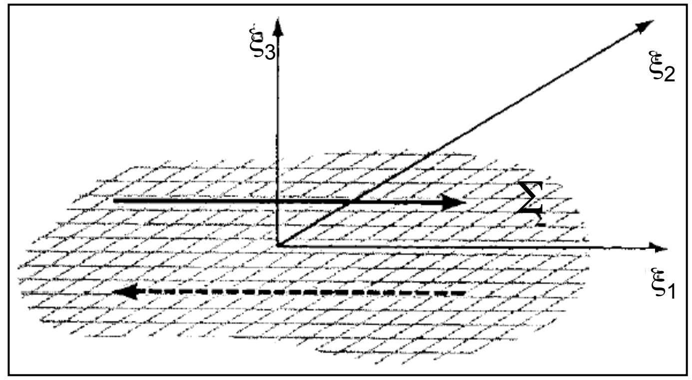
Force Equivalent for a Buried Fault

Body force equivalent to the displacement discontinuity shown in the figure below (repeated):

$$f_p(\boldsymbol{\eta}, \tau) = -\iint_\Sigma [u_1(\boldsymbol{\xi}, \tau)]\, c_{13pq} \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \boldsymbol{\xi})\, d\xi_1\, d\xi_2$$

In isotropic heterogeneous media, from

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$



we find that all c_{13pq} vanish except $c_{1313} = c_{1331} = \mu$. Hence the body-force equivalent distribution over Σ becomes:

$$f_1(\boldsymbol{\eta}, \tau) = -\iint_\Sigma \mu(\boldsymbol{\xi})\, [u_1(\boldsymbol{\xi}, \tau)]\, \delta(\eta_1 - \xi_1)\delta(\eta_2 - \xi_2) \frac{\partial}{\partial \eta_3} \delta(\eta_3)\, d\xi_1\, d\xi_2$$

$$f_2(\boldsymbol{\eta}, \tau) = 0,$$

$$f_3(\boldsymbol{\eta}, \tau) = -\iint_\Sigma \mu\, [u_1] \frac{\partial}{\partial \eta_1} \delta(\eta_1 - \xi_1)\delta(\eta_2 - \xi_2)\delta(\eta_3)\, d\xi_1\, d\xi_2.$$

Force Equivalent for a Buried Fault

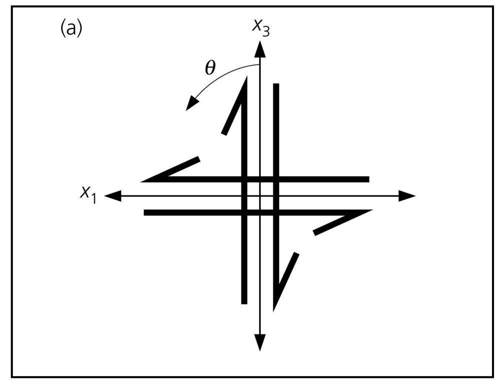
The complete body-force equivalent to fault slip consists of two parts with both canceling moment and net force:

1. A distribution of single couples (f_1) made up of forces pointing in the fault slip direction, and
2. A distribution of fault-normal single forces over Σ (f_3) with total moment cancelling the one due to f_1.

The absolute value of the total moment associated with each contribution is equal to $\mu \bar{u} A$

At great distance from the fault, wavelengths of seismic waves are much greater than the linear dimension of Σ, and their periods much longer than the source duration. The slip thus becomes localized in space and time $\bar{\mathbf{u}} A \delta(\xi_1)\delta(\xi_2) H(\tau)$ and then:

Double Couple: Point dislocation force equivalent



$$f_1(\boldsymbol{\eta}, \tau) = -M_0 \delta(\eta_1)\delta(\eta_2) \frac{\partial}{\partial \eta_3} \delta(\eta_3) H(\tau)$$

$$f_3(\boldsymbol{\eta}, \tau) = -M_0 \frac{\partial}{\partial \eta_1} \delta(\eta_1)\delta(\eta_2)\delta(\eta_3) H(\tau)$$

where M_0 is called the seismic moment:

$$M_0 = \mu \bar{u} A = \mu \times \text{ average slip } \times \text{ fault area.}$$

Moment Magnitude M_w

The total moment magnitude due to each couple of the body-force equivalent for a point dislocation is called the seismic moment M_0:

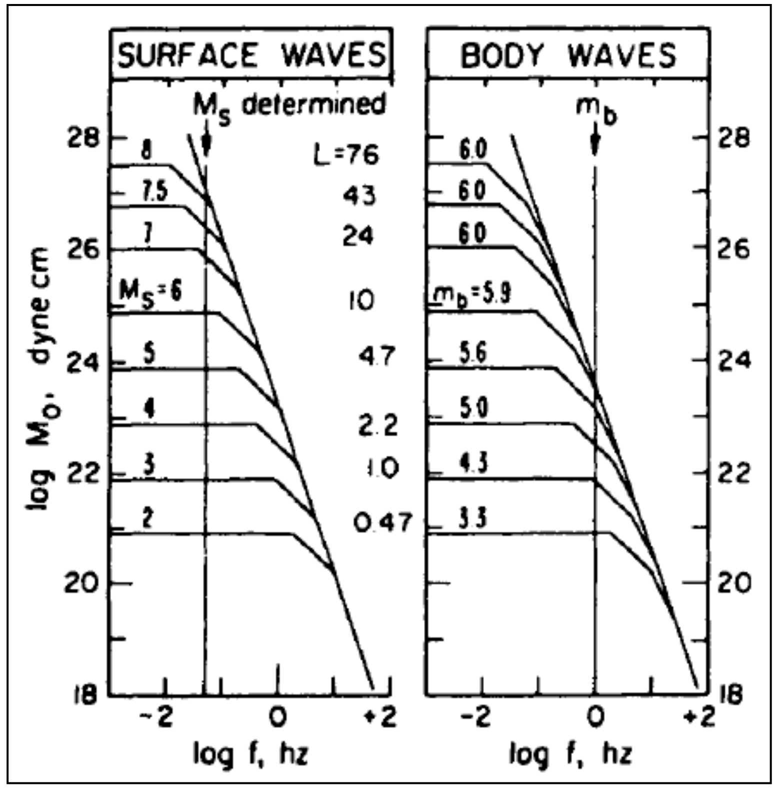
$$M_0 = \mu \bar{u} A = \mu \times \text{ average slip } \times \text{ fault area.}$$

This quantity is a measure of the source strength and does not depend on the kind of seismic wave used to determine it.

Earthquakes magnitude has been determined empirically by means of specific wave amplitudes, such as body (M_b) and surface waves (M_s).

Kanamori (1977) introduced the Moment Magnitude M_w, which is based on M_0 and approximately equal to M_s in the frequency range where the surface waves spectrum is not saturated:

$$M_w = \left(\frac{\log M_0}{1.5} \right) - 10.73.$$



The Seismic Moment Tensor

The seismic moment tensor is a quantity that depends on the source strength and orientation. It is a generalized description of body-force equivalents for seismic sources consisting of force couples and dipoles.

We start from the following displacement representation in terms of the slip on the fault and the spatial derivative of the Green tensor (repeated):

$$u_n(\mathbf{x}, t) = \int_{-\infty}^{\infty} d\tau \iint_{\Sigma} [u_i(\xi, \tau)]\, c_{ijpq} \nu_j \frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t - \tau; \xi, 0)\, d\Sigma.$$

which using the convolution symbol is written as

$$u_n(\mathbf{x}, t) = \iint_{\Sigma} [u_i]\, \nu_j c_{ijpq} * \frac{\partial}{\partial \xi_q} G_{np}\, d\Sigma.$$

Note that, if functions f(t) and g(t) are zero for t < 0, then

$$f * g = \int_0^t f(\tau) g(t - \tau)\, d\tau = \int_0^t f(t - \tau) g(\tau)\, d\tau = \int_{-\infty}^{\infty} f(\tau) g(t - \tau)\, d\tau$$

The Seismic Moment Tensor

Recall that the contribution of a time varying force f_p to the displacement field is given by

$$u_n(\mathbf{x},t) = \int_{-\infty}^{\infty} d\tau \iiint_V f_p(\boldsymbol{\eta},\tau) G_{np}(\mathbf{x}, t-\tau; \boldsymbol{\eta}, 0)\, dV(\boldsymbol{\eta})$$

If f_p is applied in the p-direction at the fault point ξ, then the n-component of displacement at (x,t) is given by the convolution $F_p * G_{np}$

On the other hand, we have that the contribution of a point dislocation to displacements is also given by the convolution of the slip on the fault and the spatial derivative of the Green tensor at the source (repeated):

$$u_n(\mathbf{x},t) = \iint_\Sigma [u_i]\, \nu_j c_{ijpq} * \frac{\partial}{\partial \xi_q} G_{np}\, d\Sigma.$$

The Seismic Moment Tensor

Recall

Displacement Discontinuity: We use the following property of the delta-function derivative to localize points of Σ within V (repeated):

$$\frac{\partial}{\partial \xi_q} G_{np}(\mathbf{x}, t-\tau; \xi, 0) = -\iiint_V \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \xi) G_{np}(\mathbf{x}, t-\tau; \boldsymbol{\eta}, 0)\, dV(\boldsymbol{\eta}),$$

so that the displacement discontinuity contributes the displacement with (repeated)

$$\int_{-\infty}^{\infty} d\tau \iiint_V \left\{ -\iint_\Sigma [u_i(\xi, \tau)]\, c_{ijpq} \nu_j \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \xi)\, d\Sigma \right\} G_{np}(\mathbf{x}, t-\tau; \boldsymbol{\eta}, 0)\, dV$$

Body-force equivalent of a displacement discontinuity

$$f_p^{[\mathbf{u}]}(\boldsymbol{\eta}, \tau) = -\iint_\Sigma [u_i(\xi, \tau)]\, c_{ijpq} \nu_j \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \xi)\, d\Sigma.$$

Force strength

The Seismic Moment Tensor

From the general representation of displacement as a function of the time varying force f_p that originate it

$$u_n(\mathbf{x}, t) = \int_{-\infty}^{\infty} d\tau \iiint_V f_p(\boldsymbol{\eta}, \tau) G_{np}(\mathbf{x}, t-\tau; \boldsymbol{\eta}, 0)\, dV(\boldsymbol{\eta})$$

Body-force equivalent of a displacement discontinuity

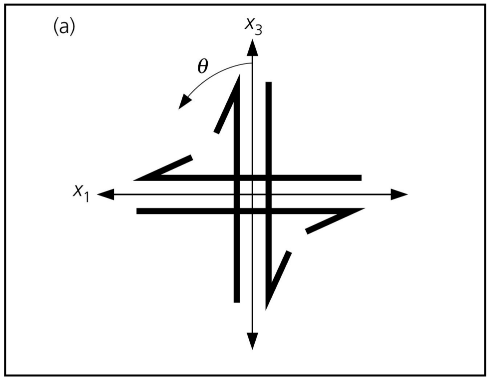
$$f_p^{[\mathbf{u}]}(\boldsymbol{\eta}, \tau) = -\iint_\Sigma [u_i(\boldsymbol{\xi}, \tau)]\, c_{ijpq} \nu_j \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \boldsymbol{\xi})\, d\Sigma.$$

Force equivalent to a point dislocation on surface $\xi_3 = 0$ with slip in the ξ_1 direction

$$f_1(\boldsymbol{\eta}, \tau) = -M_0 \delta(\eta_1)\delta(\eta_2) \frac{\partial}{\partial \eta_3} \delta(\eta_3) H(\tau)$$

$$f_3(\boldsymbol{\eta}, \tau) = -M_0 \frac{\partial}{\partial \eta_1} \delta(\eta_1)\delta(\eta_2)\delta(\eta_3) H(\tau)$$

Double Couple: Point dislocation force equivalent



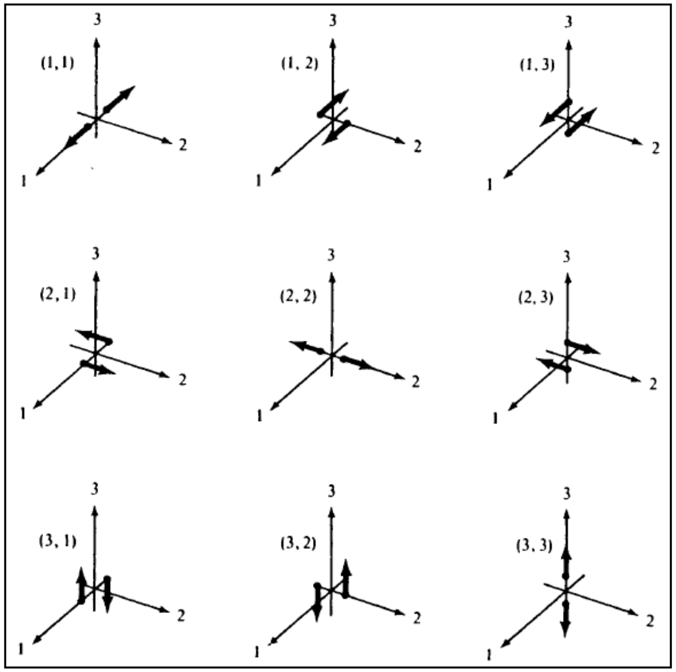
The Seismic Moment Tensor

For a displacement discontinuity (i.e. fault slip or opening), the representation formula depends on the spatial derivatives of the Green function G_{np}

$$u_n(\mathbf{x}, t) = \iint_\Sigma [u_i]\, \nu_j c_{ijpq} * \frac{\partial}{\partial \xi_q} G_{np}\, d\Sigma.$$

that, as previously demonstrated, correspond to a single force couple each, with arm in the ξ_q-direction. The sum over index q tells us that each displacement component at x is the contribution of a sum of force couples and dipoles distributed over Σ.

Since the integrand is the n-component of the displacement at x due to couples at ξ, it follows that $[u_i]\, \nu_j c_{ijpq}$ is the strength of the (q,p) couple, which has units of moment per unit area. The moment contribution of each fault unit area is the strength weighted by the infinitesimal surface area $d\Sigma$.



The Seismic Moment Tensor

We thus define the time-dependent components of the moment density tensor to be:

$$m_{pq} = [u_i]\, \nu_j c_{ijpq}$$

In terms of this tensor, the representation theorem for displacement at x due to general displacement discontinuity [u(ξ,τ)] across Σ is:

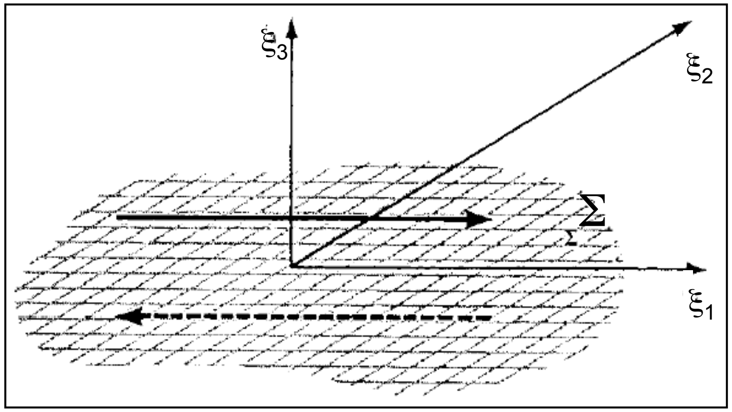
$$u_n(\mathbf{x}, t) = \iint_{\Sigma} m_{pq} * G_{np,q}\, d\Sigma.$$

In an isotropic medium with displacement discontinuity without fault opening (i.e. slip or shear dislocation), the moment density tensor becomes

$$m_{pq} = \mu \left(\nu_p [u_q] + \nu_q [u_p] \right).$$

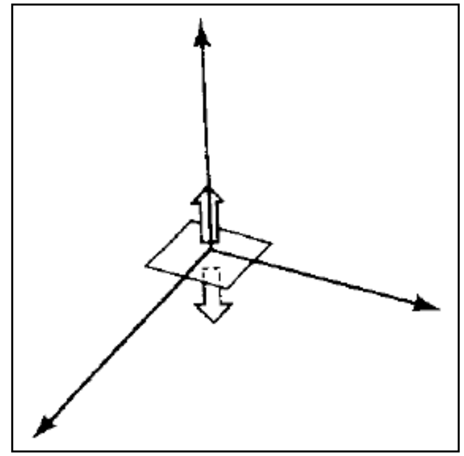
If Σ lies in the $\xi_3 = 0$ plane with slip only in the ξ_1-direction (figure), the moment density consists of the force double-couple:

$$\mathbf{m} = \begin{pmatrix} 0 & 0 & \mu [u_1(\xi, \tau)] \\ 0 & 0 & 0 \\ \mu [u_1(\xi, \tau)] & 0 & 0 \end{pmatrix},$$



The Seismic Moment Tensor

If Σ lies in the ξ_3 = 0 plane and $[u_3]$ is the only nonzero displacement component (figure), the moment density consists of three force dipoles:



$$\mathbf{m} = \begin{pmatrix} \lambda\,[u_3(\xi,\tau)] & 0 & 0 \\ 0 & \lambda\,[u_3(\xi,\tau)] & 0 \\ 0 & 0 & (\lambda+2\mu)\,[u_3(\xi,\tau)] \end{pmatrix}$$

In observational seismology we often analyze seismic waves with very long periods for which the whole Σ is effectively a point source. So by integrating the contribution of every single fault unit area, the representation of displacement reads

$$u_n(\mathbf{x},t) = M_{pq} * G_{np,q}$$

where the moment tensor, M_{pq}, is thus given by

$$M_{pq} = \iint_\Sigma m_{pq}\, d\Sigma = \iint_\Sigma [u_i]\, \nu_j c_{ijpq}\, d\Sigma,$$

The Seismic Moment Tensor

The seismic moment tensor may be expressed as

$$M_{kj} = \mu A(D_k \mathbf{v}_j + D_j \mathbf{v}_k).$$

where A is the total fault area and D_i the slip vector. Its components are:

$$M_{11} = -M_0(\sin\delta\cos\lambda\sin 2\phi_f + \sin 2\delta\sin\lambda\sin^2\phi_f)$$

$$M_{22} = M_0(\sin\delta\cos\lambda\sin 2\phi_f - \sin 2\delta\sin\lambda\cos^2\phi_f)$$

$$M_{33} = M_0(\sin 2\delta\sin\lambda) = -(M_{11} + M_{22})$$

$$M_{12} = M_0(\sin\delta\cos\lambda\cos 2\phi_f + \tfrac{1}{2}\sin 2\delta\sin\lambda\sin 2\phi_f)$$

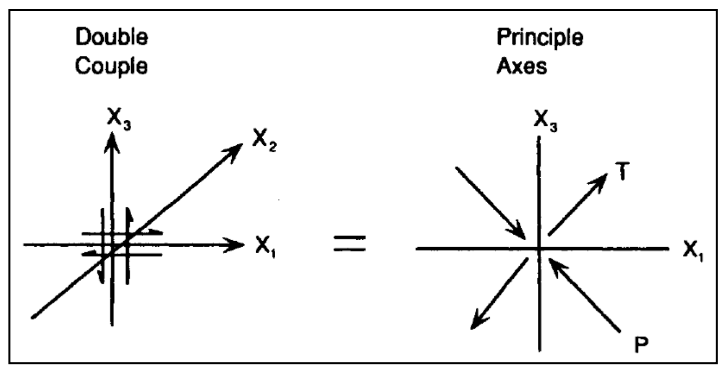
$$M_{13} = -M_0(\cos\delta\cos\lambda\cos\phi_f + \cos 2\delta\sin\lambda\sin\phi_f)$$

$$M_{23} = -M_0(\cos\delta\cos\lambda\sin\phi_f - \cos 2\delta\sin\lambda\cos\phi_f).$$

δ = dip
λ = rake
ϕ= strike
$M_0 = \mu AD$

Since the moment tensor is symmetric, it may be diagonalized by rotating it into a principal-axis system.

The rotated tensor components correspond to the eigenvalues of the moment tensor, and the associated eigenvectors give the directions of the tensional (T), intermediate (B) and compressional (P) stress axis.



The Seismic Moment Tensor

The diagonalized moment tensor may be decomposed into two separate tensors, namely the isotropic and deviatoric moment tensors, so that

$$\mathbf{M} = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \mathrm{tr}(\mathbf{M}) & 0 & 0 \\ 0 & \mathrm{tr}(\mathbf{M}) & 0 \\ 0 & 0 & \mathrm{tr}(\mathbf{M}) \end{bmatrix} + \begin{bmatrix} M_1^1 & 0 & 0 \\ 0 & M_2^1 & 0 \\ 0 & 0 & M_3^1 \end{bmatrix},$$

Where $\mathrm{tr}(M) = M_1 + M_2 + M_3$ is the trace of M, and the remaining terms M_i are the deviatoric eigenvalues of M.

The isotropic moment tensor components describe the volume change of the medium due to either an explosion or implosion. Most shearing sources appear to have little isotropic component, so often, when determining their moment tensor, seismologists assume that $\mathrm{tr}(M) = 0$.

The deviatoric moment tensor may be decomposed in different ways: three vector dipoles, three compensated linear vector dipoles (CLVDs), a double couple and a CLVD, etc.