Elastic Waves From a Double-Couple Point Source

1. The double-couple wavefield solution
2. Radiation pattern of a point dislocation
3. Point dislocations in nature

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The Double-Couple Wavefield Solution

The representation of displacement u_i due to body-force f_i, fault dislocation $[u_i]$ and discontinuity of tractions across Σ (i.e. the fault) $[T_i]$ within an unbounded medium reads

$$u_n(x,t) = \int_{-\infty}^{\infty} d\tau \iiint_V f_p(\boldsymbol{\eta},\tau) G_{np}(\mathbf{x}, t-\tau; \boldsymbol{\eta}, 0)\, dV(\boldsymbol{\eta})$$
$$+ \int_{-\infty}^{\infty} d\tau \iint_\Sigma \Big\{ \big[u_i(\xi,\tau)\big] c_{ijpq} \nu_j G_{np,q}(\mathbf{x}, t-\tau; \xi, 0)$$
$$- \Big[T_p(\mathbf{u}(\xi,\tau), \boldsymbol{\nu})\Big] G_{np}(\mathbf{x}, t-\tau; \xi, 0)\Big\}\; d\Sigma(\xi).$$

Displacement due to body-force $\mathbf{f}(\mathbf{x},t) = \mathbf{F}(t)\delta(\mathbf{x}-\xi)$ may thus be written

$$u_n(x,t) = \int_{-\infty}^{\infty} d\tau \iiint_V f_p(\boldsymbol{\eta},\tau) G_{np}(\mathbf{x}, t-\tau; \boldsymbol{\eta}, 0)\, dV(\boldsymbol{\eta}) = F_p * G_{np}$$

The Double-Couple Wavefield Solution

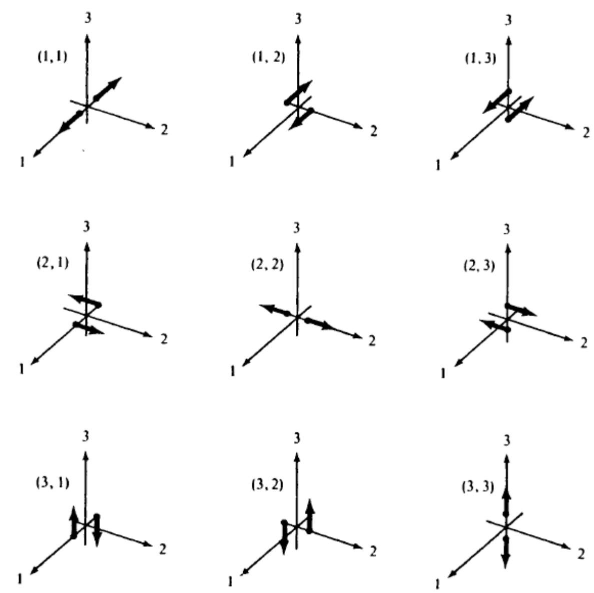
Which is explicitly given by the Stokes' solution for the Green function:

$$F_p * G_{np} = \frac{1}{4\pi\rho}(3\gamma_n\gamma_p - \delta_{np})\frac{1}{r^3}\int_{r/\alpha}^{r/\beta}\tau F_p(t-\tau)\,d\tau$$
$$+ \boxed{\frac{1}{4\pi\rho\alpha^2}\gamma_n\gamma_p\frac{1}{r}F_p\left(t-\frac{r}{\alpha}\right)} - \frac{1}{4\pi\rho\beta^2}(\gamma_n\gamma_p - \delta_{np})\frac{1}{r}F_p\left(t-\frac{r}{\beta}\right)$$

where r is the distance between the point-source and receiver: $r = |\mathbf{x} - \xi|$ and γ_i the direction cosines $\gamma_i = (x_i - \xi_i)/r$ given by their relative position that determines the radiation pattern. Note the summation over index *p*.

Our goal is find out the way to express the displacement due to a double-couple force equivalent (i.e. a point dislocation) by means of the Stokes' solution, which corresponds to the displacement due to a single body-force.

Let us first analyze the P-wave far-field displacement.



The Double-Couple Wavefield Solution

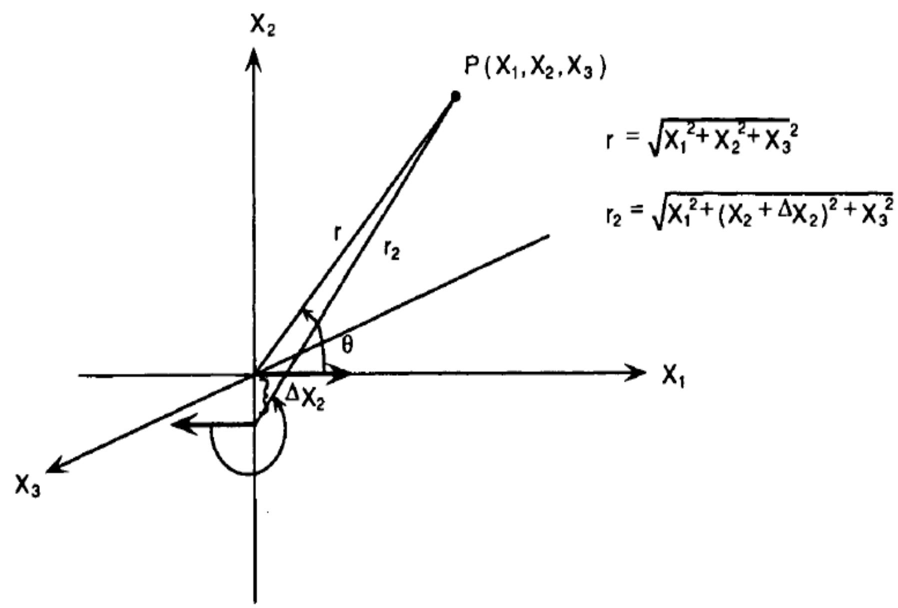
The P-wave far-field contribution due to a single body-force *F* is given by:

$$u_i^P = \frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_j\frac{1}{r}F\left(t-\frac{r}{\alpha}\right)$$

Denoting the time history of the body-force as h(t), the displacement field due to a single couple of forces disposed as shown in the figure, is the addition of both contributions:

$$u_i^c = \frac{1}{4\pi\rho\alpha^2}\left[\gamma_i\gamma_1\frac{h(t-(r/\alpha))}{r} + \gamma_i^{F_2}\gamma_{-1}^{F_2}\frac{h(t-(r_2/\alpha))}{r_2}\right].$$

A couple of forces with moment parallel to the x_3 axis and arm parallel to the x_2 axis.



The Double-Couple Wavefield Solution

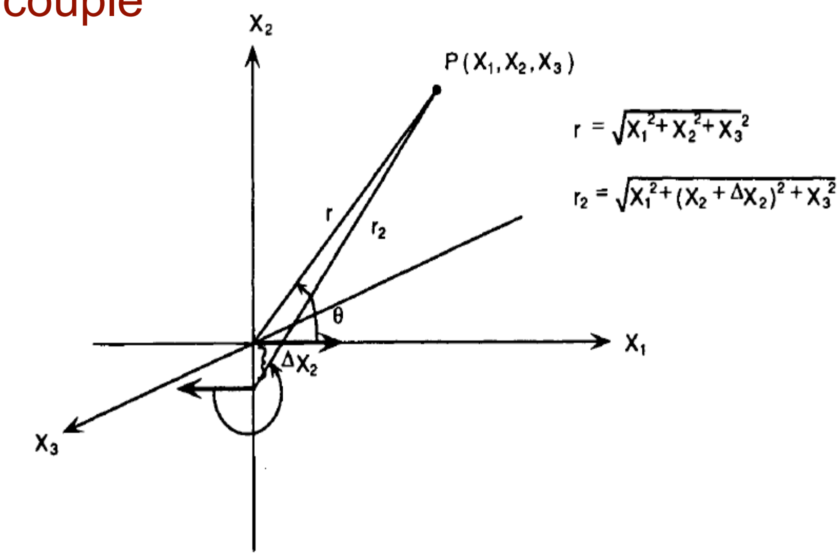
$$u_i^c = \frac{1}{4\pi\rho\alpha^2}\left[\gamma_i\gamma_1\frac{h(t-(r/\alpha))}{r} + \gamma_i^{F_2}\gamma_{-1}^{F_2}\frac{h(t-(r_2/\alpha))}{r_2}\right].$$

Since Δr << r (i.e. $1/r \approx 1/r_2$) the direction cosines satisfy

$$\gamma_i^{F_2} = \gamma_i - \frac{\Delta x_2}{r}\delta_{2j} \quad \text{and} \quad \gamma_{-1}^{F_2} \approx -\gamma_i$$

So the displacement field from the force couple may be approximated as

$$u_i^c = \frac{\gamma_1}{4\pi\rho\alpha^2}\left[\gamma_i\frac{h(t-(r/\alpha))}{r} - \left(\gamma_i - \frac{\Delta x_2}{r}\delta_{2j}\right)\frac{h(t-(r_2/\alpha))}{r_2}\right]$$



The Double-Couple Wavefield Solution

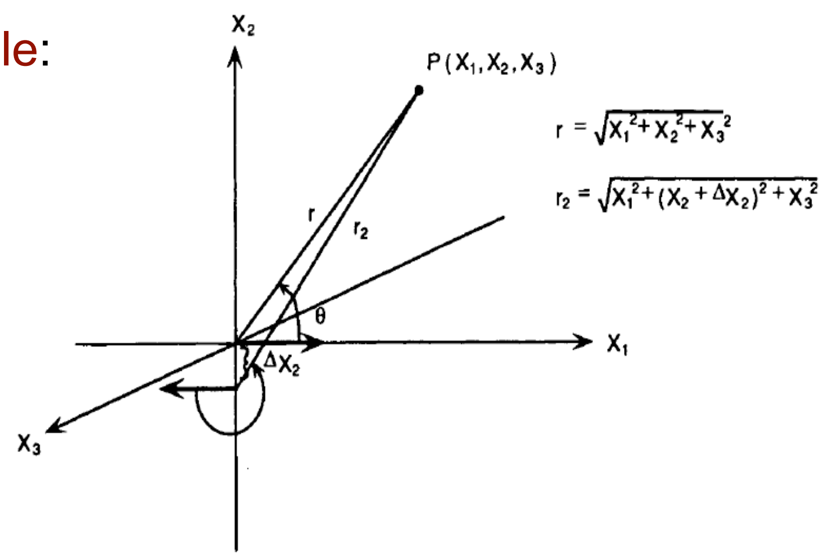
$$u_i^c = \frac{\gamma_1}{4\pi\rho\alpha^2}\left[\gamma_i \frac{h(t-(r/\alpha))}{r} - \left(\gamma_i - \frac{\Delta x_2}{r}\delta_{2j}\right)\frac{h(t-(r_2/\alpha))}{r_2}\right]$$

By taking $h(t-(r_2/\alpha)) = h(t-(r/\alpha)-(\Delta r/\alpha))$, expanding functions h(t) in Taylor series and assuming that $1/r \approx 1/r_2$

$$u_i^c = \frac{\gamma_1}{4\pi\rho\alpha^2}\left\{\gamma_i\left[\frac{h(t-(r/\alpha))}{r}\right] - \gamma_i\left[\frac{h(t-(r/\alpha))}{r} - \frac{\Delta r}{\alpha}\frac{\dot{h}(t-(r/\alpha))}{r}\right]\right.$$
$$\left. + \frac{\Delta x_2}{r}\delta_{2j}\left[\frac{h(t-(r/\alpha))}{r} - \frac{\Delta r}{\alpha}\frac{\dot{h}(t-(r/\alpha))}{r}\right]\right\}$$

The displacement field from the force couple:

$$u_i^c = \frac{\gamma_1}{4\pi\rho\alpha^2}\left\{\gamma_i \frac{\Delta r \dot{h}(t-(r/\alpha))}{\alpha r}\right.$$
$$\left. + \frac{\Delta x_2 \delta_{2j}}{r^2}\left[h\left(t-\frac{r}{\alpha}\right) - \frac{\Delta r}{\alpha}\dot{h}\left(t-\frac{r}{\alpha}\right)\right]\right\}.$$



The Double-Couple Wavefield Solution

Dismissing both terms that decay as $1/r^2$ (i.e. near-field terms) compared to the far-field term, and since

$$\Delta r \approx \frac{\partial r}{\partial x_2} \Delta x_2 = \gamma_2 \Delta x_2$$

The far-field displacement due to the single force-couple becomes:

$$u_i^c = \frac{\gamma_1 \gamma_2 \gamma_i}{4\pi\rho\alpha^3} \left[\boxed{ \frac{\Delta x_2}{r} \dot{h}\left(t - \frac{r}{\alpha}\right) } \right].$$

Now, if we consider that $\Delta x_2 \to 0$ and $h \to \infty$ such that $\Delta x_2 h \to M$, which is the moment of the force couple, then

$$M\left(t - \frac{r}{\alpha}\right) = \lim_{\substack{\Delta x_2 \to 0 \\ h \to \infty}} \Delta x_2 h\left(t - \frac{r}{\alpha}\right) \Rightarrow \dot{M}\left(t - \frac{r}{\alpha}\right) = \lim_{\substack{\Delta x_2 \to 0 \\ h \to \infty}} \boxed{\Delta x_2 \dot{h}\left(t - \frac{r}{\alpha}\right)}.$$

So the solution for the couple is:

$$u_i^c = \frac{\gamma_1 \gamma_2 \gamma_i}{4\pi\rho\alpha^3} \frac{\dot{M}(t - (r/\alpha))}{r}.$$

The Double-Couple Wavefield Solution

The solution for the couple is: $$u_i^c = \frac{\gamma_1\gamma_2\gamma_i}{4\pi\rho\alpha^3}\frac{\dot{M}(t-(r/\alpha))}{r}.$$

By doing the same analysis for a couple with an orientation parallel to the x_2 axis, and if we offset F_2 by Δx_1 in the negative x_2 direction we would get an identical result because of the symmetry of the products of γ_i.

The summation of displacement contributions due to both force couples (i.e. a double-couple) shows that the P-wave far-field produced by a point dislocation is proportional to the moment rate time history (i.e. fault slip rate):

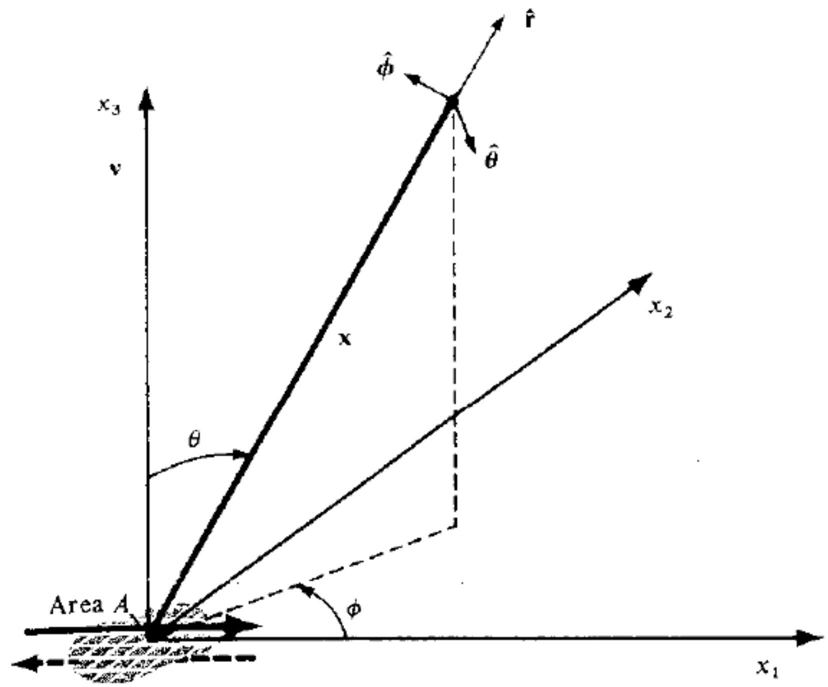
$$u_i^{\mathrm{Dc}} = 2\left[\frac{\gamma_1\gamma_2\gamma_i}{4\pi\rho\alpha^3}\frac{\boxed{\dot{M}(t-(r/\alpha))}}{r}\right].$$

Our initial representation theorem tell us that the complete far-field displacements produced by a point dislocation is given by the following convolution

$$u_n(\mathbf{x},t) = \iint_\Sigma [u_i]\,\nu_j c_{ijpq} * \frac{\partial}{\partial\xi_q} G_{np}\,d\Sigma = \boxed{M_{pq} * G_{np,q}}$$

The Double-Couple Wavefield Solution

The complete displacement field u_i in spherical coordinates r, θ and ϕ (see figure) due to a point dislocation (i.e. a double-couple) in an unbounded homogeneous space, with the time-dependent seismic moment equal to $M_0(t) = \mu \bar{u}(t) A$ is



$$\mathbf{u}(\mathbf{x}, t) = \boxed{\frac{1}{4\pi\rho} \mathbf{A}^N \frac{1}{r^4} \int_{r/\alpha}^{r/\beta} \tau M_0(t-\tau)\, d\tau} \quad \text{Near-field}$$

$$+ \boxed{\frac{1}{4\pi\rho\alpha^2} \mathbf{A}^{IP} \frac{1}{r^2} M_0\left(t - \frac{r}{\alpha}\right)} + \boxed{\frac{1}{4\pi\rho\beta^2} \mathbf{A}^{IS} \frac{1}{r^2} M_0\left(t - \frac{r}{\beta}\right)} \quad \text{Intermediate-field}$$

$$+ \boxed{\frac{1}{4\pi\rho\alpha^3} \mathbf{A}^{FP} \frac{1}{r} \dot{M}_0\left(t - \frac{r}{\alpha}\right)} + \boxed{\frac{1}{4\pi\rho\beta^3} \mathbf{A}^{FS} \frac{1}{r} \dot{M}_0\left(t - \frac{r}{\beta}\right)}, \quad \text{Far-field}$$

The Double-Couple Wavefield Solution

Point-dislocation complete wavefield solution in a homogeneous fullspace:

$$\mathbf{u}(\mathbf{x}, t) = \frac{1}{4\pi\rho}\mathbf{A}^{N}\frac{1}{r^4}\int_{r/\alpha}^{r/\beta} \tau M_0(t-\tau)\, d\tau$$

$$+ \frac{1}{4\pi\rho\alpha^2}\mathbf{A}^{IP}\frac{1}{r^2}M_0\left(t - \frac{r}{\alpha}\right) + \frac{1}{4\pi\rho\beta^2}\mathbf{A}^{IS}\frac{1}{r^2}M_0\left(t - \frac{r}{\beta}\right)$$

$$+ \frac{1}{4\pi\rho\alpha^3}\mathbf{A}^{FP}\frac{1}{r}\dot{M}_0\left(t - \frac{r}{\alpha}\right) + \frac{1}{4\pi\rho\beta^3}\mathbf{A}^{FS}\frac{1}{r}\dot{M}_0\left(t - \frac{r}{\beta}\right),$$

with the radiation pattern given by the following coefficients, which only depend on two angles, θ and ϕ, defining the source-receiver position with respect to the slip:

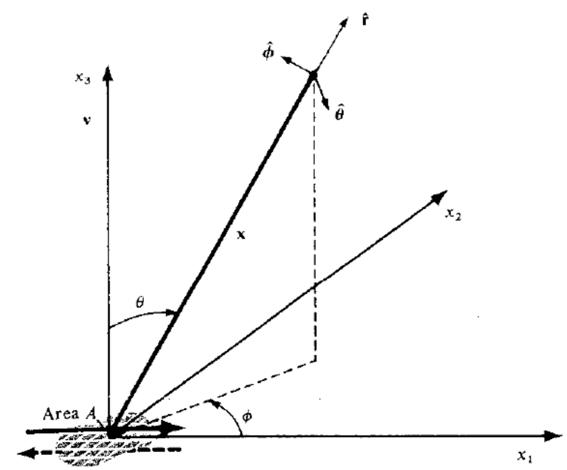
$$\mathbf{A}^{N} = 9 \sin 2\theta \cos\phi\, \hat{\mathbf{r}} - 6(\cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}})$$

$$\mathbf{A}^{IP} = 4 \sin 2\theta \cos\phi\, \hat{\mathbf{r}} - 2(\cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}})$$

$$\mathbf{A}^{IS} = -3 \sin 2\theta \cos\phi\, \hat{\mathbf{r}} + 3(\cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}})$$

$$\mathbf{A}^{FP} = \sin 2\theta \cos\phi\, \hat{\mathbf{r}}$$

$$\mathbf{A}^{FS} = \cos 2\theta \cos\phi\, \hat{\boldsymbol{\theta}} - \cos\theta \sin\phi\, \hat{\boldsymbol{\phi}}.$$



The Double-Couple Wavefield Solution

The final displacement (i.e. static field) due to a point dislocation source is obtained throughout the limit of $\dot{M}_0(t-\tau)$, $M_0(t-\tau)$ and $\int_{r/\alpha}^{r/\beta} \tau M_0(t-\tau)\,d\tau$ as $t \to \infty$. The results is

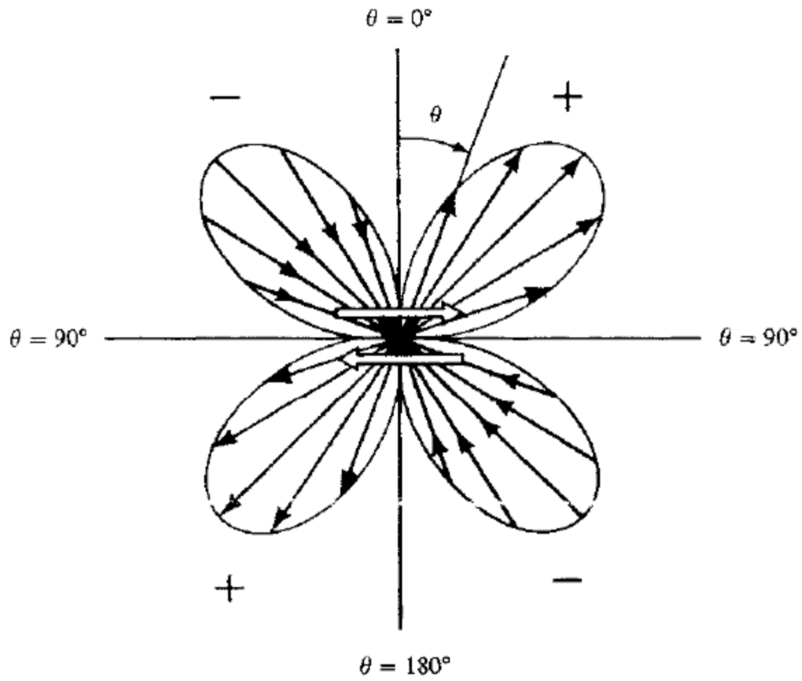
$$\mathbf{u}(\mathbf{x},\infty) = \frac{M_0(\infty)}{4\pi\rho r^2}\left[\mathbf{A}^N\left(\frac{1}{2\beta^2} - \frac{1}{2\alpha^2}\right) + \frac{\mathbf{A}^{IP}}{\alpha^2} + \frac{\mathbf{A}^{IS}}{\beta^2}\right]$$

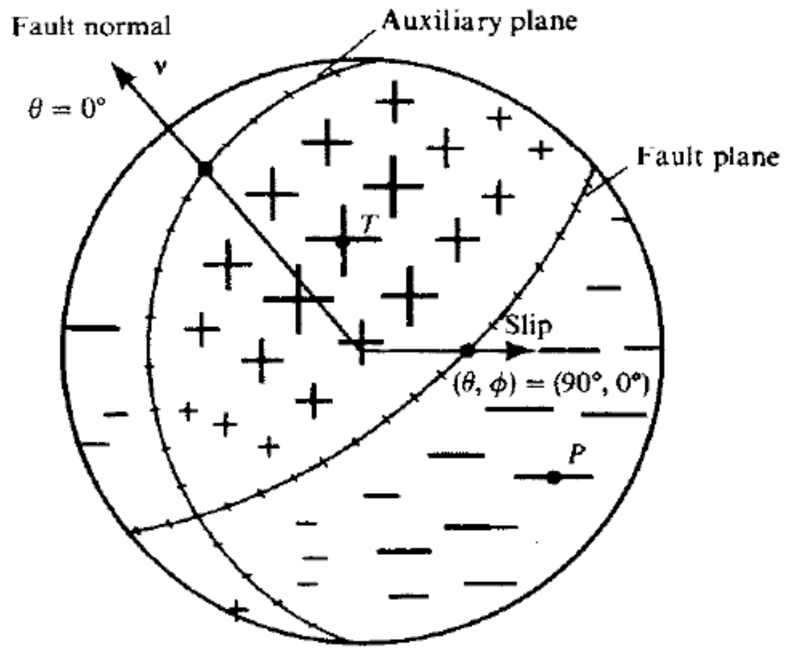
$$= \frac{M_0(\infty)}{4\pi\rho r^2}\left[\frac{1}{2}\left(\frac{3}{\beta^2} - \frac{1}{\alpha^2}\right)\sin 2\theta\cos\phi\,\hat{\mathbf{r}} + \frac{1}{\alpha^2}(\cos 2\theta\cos\phi\,\hat{\boldsymbol{\theta}} - \cos\theta\sin\phi\,\hat{\boldsymbol{\phi}})\right]$$

which attenuates along any given direction θ and ϕ as r^{-2}.

Radiation Pattern from a Point Dislocation

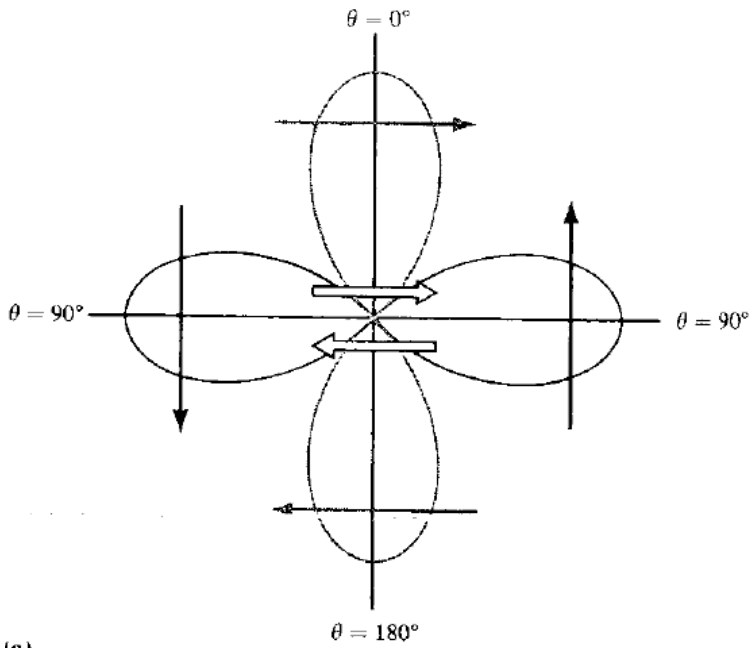
Radiation pattern for the radial component of displacement, all terms combined. There are two nodal planes where this component disappears: the fault plane and the auxiliary plane. Displacement is maximum along two lines T and P.

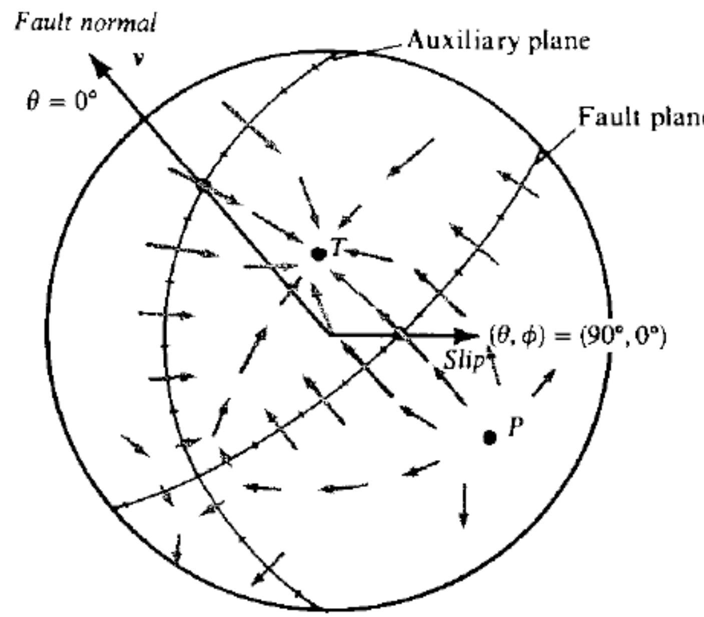




Radiation Pattern from a Point Dislocation

Radiation pattern for the transverse component of displacement, all terms combined. There are two directions where this component is maximum: lines parallel and perpendicular to the slip vector. There are three directions where it disappears: along the two lines T and P and along the their perpendicular direction (i.e. intersection of the nodal planes for the radial displacement).

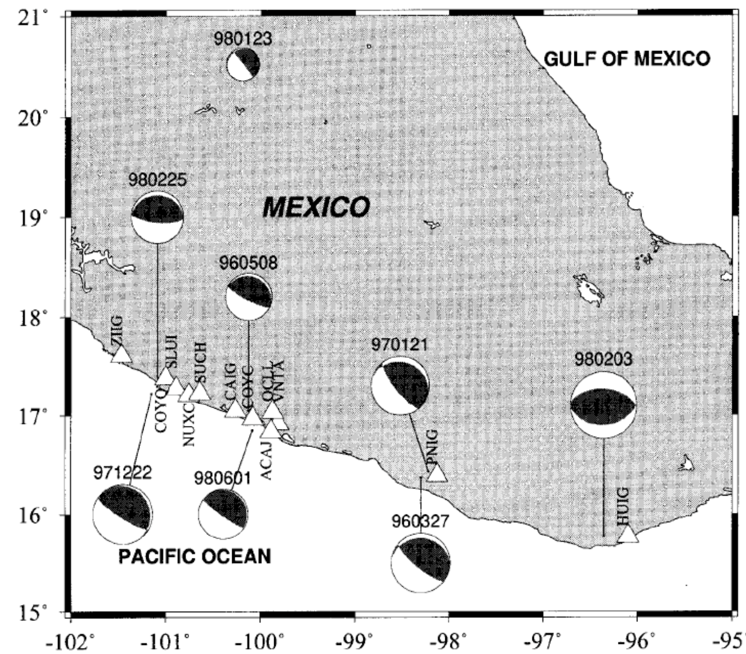


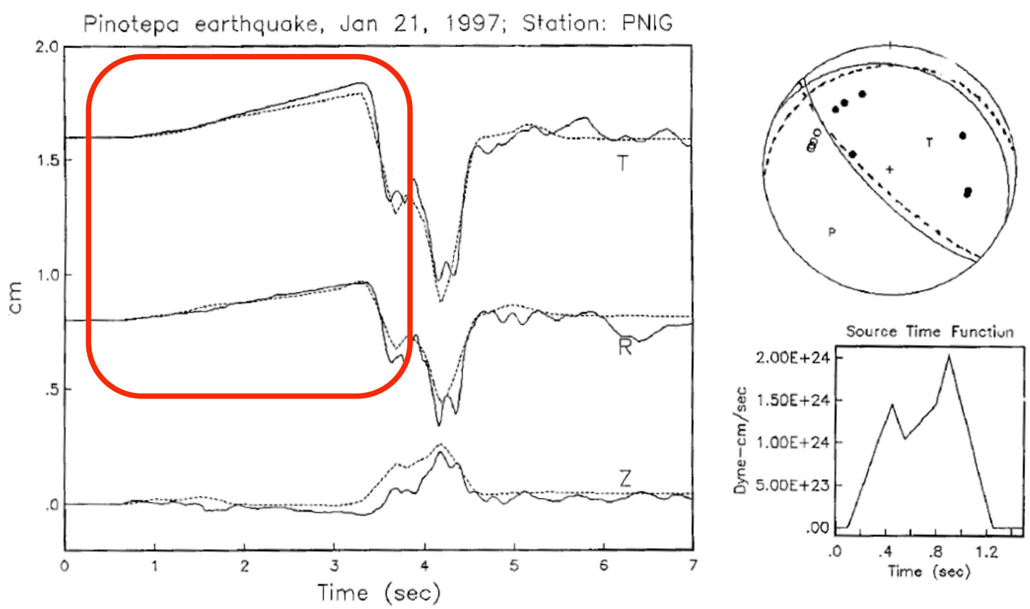


Point Dislocations in Nature

Two moderate subduction earthquakes (both Mw=5.4) were recorded at the near-source VBB station of PNIG.

By integrating twice the strong motion records (accelerograms) both location and focal mechanism (i.e. moment tensor) where estimated from displacements.

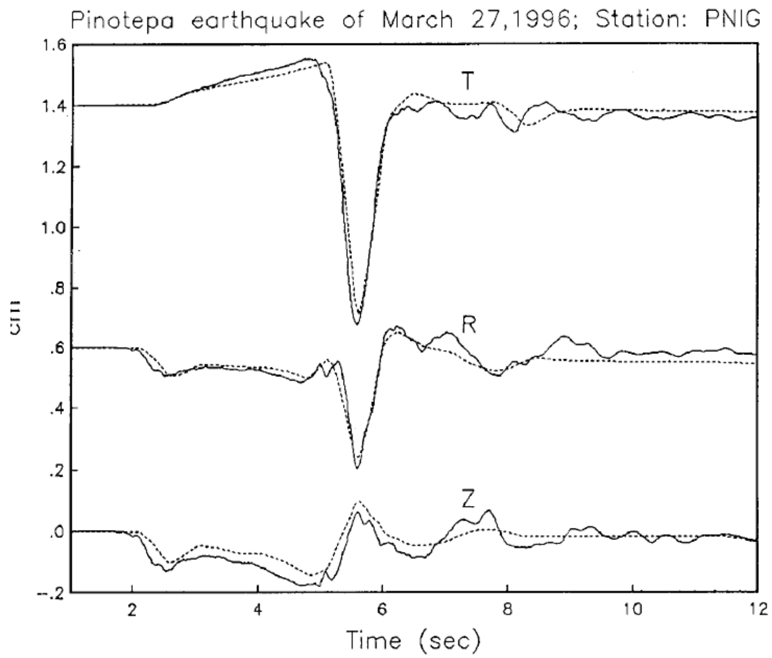


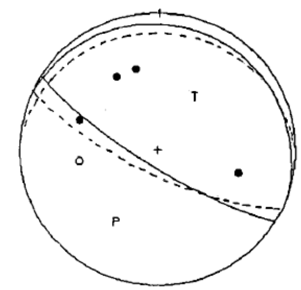


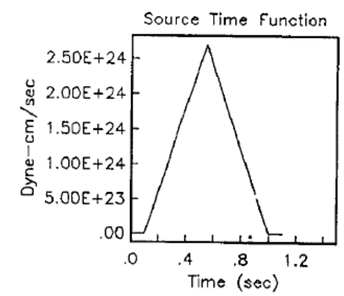
Notice the displacement ramp between the P and S arrivals associated with the near field term in both the observed (solid) and synthetic (dashed) seismograms (Pacheco and Singh, Geof. Int., 1998).

Point Dislocations in Nature

Notice again the displacement ramp between the P and S arrivals associated with the near field term in both the observed (solid) and synthetic (dashed) seismograms (Pacheco and Singh, Geof. Int., 1998).







Event	Moment (N-m)	Strike (°)	Dip (°)	Rake (°)	Source-time function
960327	1.2×10^{17}	291	10	80	Isosceles triangle base of 0.9 sec. A small event Mw 4.1 preceded the mainshock by 0.18 sec.
970121	1.2×10^{17}	296	18	70	Two isosceles triangles, each with base of 0.35 sec., separated by 0.48 sec and seismic moments of 4.8×10^{16} and 7.2×10^{16} N-m.

The focal mechanism (FM) and scalar moment M_0 where estimated since there exist also expressions for the radiation pattern coefficients in term of the three angles defining the FM (see Aki and Richards, 2002).