Finite Fault Far-Field Seismic Radiation

1. Unidirectional source rupture model
2. Finite fault far-field radiation
3. Source directivity effects

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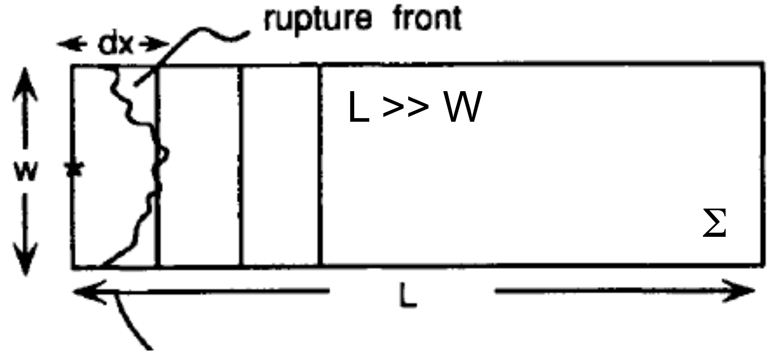
Fault Model with Unidirectional Rupture Propagation

Consider a rectangular fault plane with length L and width W so that L >> W. Rupture initiates at one extremity of the fault and propagates along L with velocity v_r.

Thus, the fault plane may be thought as a linear succession of point dislocations breaking subsequently with a time delay $dt=dx/v_r$.



Neglecting body forces and stress discontinuities across the fault, Σ, the displacement field due to a point dislocation $[u_i(\xi,\tau)]$ on the fault has the components

$$u_i(\mathbf{x}, t) = \int_{-\infty}^{\infty} d\tau \iint_{\Sigma} [u_j(\xi, \tau)] c_{jkpq} G_{ip,q}(\mathbf{x}, t; \xi, \tau) \nu_k \, d\Sigma(\xi),$$

In a homogeneous, isotropic, unbounded medium, the Stokes' solution gives an explicit form of the Green function, G_{ip}.

Fault Model with Unidirectional Rupture Propagation

Taking the body force in the Stokes' displacement solution to be a unit impulse in time, and thanks to the Green function time reciprocity, it follows that

$$G_{ip}(\mathbf{x}, t; \xi, \tau) = \frac{1}{4\pi\rho}(3\gamma_i\gamma_p - \delta_{ip})\frac{1}{r^3}\int_{r/\alpha}^{r/\beta} t'\,\delta(t-\tau-t')\,dt'$$

Far-field terms

$$+\frac{1}{4\pi\rho\alpha^2}\gamma_i\gamma_p\frac{1}{r}\delta\left(t-\tau-\frac{r}{\alpha}\right)$$

$$-\frac{1}{4\pi\rho\beta^2}(\gamma_i\gamma_p-\delta_{ip})\frac{1}{r}\delta\left(t-\tau-\frac{r}{\beta}\right),$$

where γ_i is the director cosine of the vector that goes from the source point ξ to the receiver point x, and $r=|x-\xi|$ is the distance between those two points.

If the receiver position x is sufficiently far from all points ξ on the fault surface Σ, then only the far-field terms in the Green function are significant.

By inputting these terms into the representation formula for the point dislocation, and after carrying out the time integration we obtain the corresponding far-field displacement.

Fault Model with Unidirectional Rupture Propagation

Therefore, the far-field displacement due to a point dislocation within a isotropic, unbounded, homogeneous space is

$$u_i(\mathbf{x}, t) = -\frac{1}{4\pi\rho\alpha^2}\frac{\partial}{\partial x_q}\iint_\Sigma c_{jkpq}\frac{\gamma_i\gamma_p}{r}\left[u_j\left(\xi, t-\frac{r}{\alpha}\right)\right]\nu_k\,d\Sigma$$

$$+\frac{1}{4\pi\rho\beta^2}\frac{\partial}{\partial x_q}\iint_\Sigma c_{jkpq}\left(\frac{\gamma_i\gamma_p-\delta_{ip}}{r}\right)\left[u_j\left(\xi, t-\frac{r}{\beta}\right)\right]\nu_k\,d\Sigma,$$

where the substitution of $\partial/\partial\xi_q = -\partial/\partial x_q$ has been done since γ_i and r are only dependent on the difference between x and ξ.

Carrying out the differentiation with respect to x_q, noting that $\partial r/\partial x_q$ is equal to γ_q, and taking only the first order terms of the expansion in Taylor series of the dislocation time functions, which decrease as 1/r, we obtain

$$u_i(\mathbf{x}, t) = \iint_\Sigma \frac{c_{jkpq}}{4\pi\rho\alpha^3 r}\gamma_i\gamma_p\left[\dot{u}_j\left(\xi, t-\frac{r}{\alpha}\right)\right]\gamma_q\nu_k\,d\Sigma$$

$$-\iint_\Sigma \frac{c_{jkpq}}{4\pi\rho\beta^3 r}(\gamma_i\gamma_p-\delta_{ip})\left[\dot{u}_j\left(\xi, t-\frac{r}{\beta}\right)\right]\gamma_q\nu_k\,d\Sigma.$$

Fault Model with Unidirectional Rupture Propagation

Notice that the P and S-wave far-field displacements are now expressed in terms of the slip velocity on the fault plane so that both quantities are proportional.

Using the principle of linear superposition for the P-wave far-field displacement (i.e. integrating over Σ) due to the subsequent point dislocations (with time delay $\Delta t = x/v_r$) in our linear fault model:

$$u_r(r,t) = \frac{R_i^P \mu}{4\pi\rho\alpha^3} w \sum_{i=1}^{N} \frac{\dot{D}_i}{r_i}(t - \Delta t_i)\, dx.$$

where the displacement rate discontinuity (i.e. fault slip rate) is now denoted as $\dot{D}_i$, the radiation pattern factors are regrouped in the terms R_i^P, *w* is the fault width and N is the amount of subfaults over Σ.

If the receiver is quite far from the fault, both the distances r_i and radiation patterns R_i^P may be approximated constant along the fault, and since v_r and D_i are the same along the fault:

$$u_r(r,t) = \frac{R^P \mu}{4\pi\rho\alpha^3} \frac{w}{r} \sum_{i=1}^{N} \dot{D}\left(t - \frac{x}{v_r}\right) dx.$$

Fault Model with Unidirectional Rupture Propagation

Using the shift property of the delta function

$$\dot{D}\left(t-\frac{x}{v_r}\right)=\dot{D}(t)*\delta\left(t-\frac{x}{v_r}\right)$$

and taking the limit of the summation as dx tends to zero, we obtain the integral equation

$$u_r(r,t)=\frac{R^P\mu}{4\pi\rho\alpha^3}\frac{w}{r}\int_0^x\dot{D}(t)*\delta\left(t-\frac{x}{v_r}\right)dx$$

where x is the length of the fault. Since the slip history is constant along the fault and considering the change of variable $z=t-(x/v_r)$, we have that $dx=(dx/dz)\,dz=-v_r\,dz$, and then

$$\int_0^x\delta\left(t-\frac{x}{v_r}\right)dx=\int_t^{t-(x/v_r)}-v_r\,\delta(z)\,dz.$$

Fault Model with Unidirectional Rupture Propagation

Thus

$$u_r(r,t) = \frac{R^P \mu w}{4\pi\rho\alpha^3 r}\dot{D}(t) * v_r H(z)\Big|_{t-(x/v_r)}^{t}$$

$$= \frac{R^P \mu w}{4\pi\rho\alpha^3 r} v_r \dot{D}(t) * \left[H(t) - H\left(t - \frac{x}{v_r}\right)\right]$$

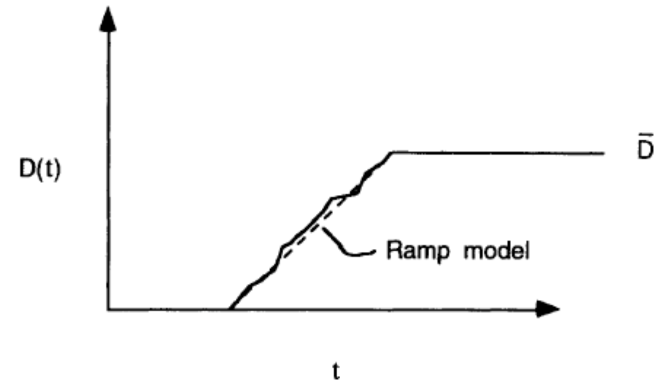
where H(t) is the heaviside step function, which is zero for t < 0 and one elsewhere. Denoting the total rupture time as $\tau_c = x/v_r$ and the boxcar function with duration τ_c as $B(t\,;\,\tau_c)$, we have

$$u_r(r,t) = \frac{R^P \mu w}{4\pi\rho\alpha^3 r} v_r \dot{D}(t) * B(t;\tau_c).$$

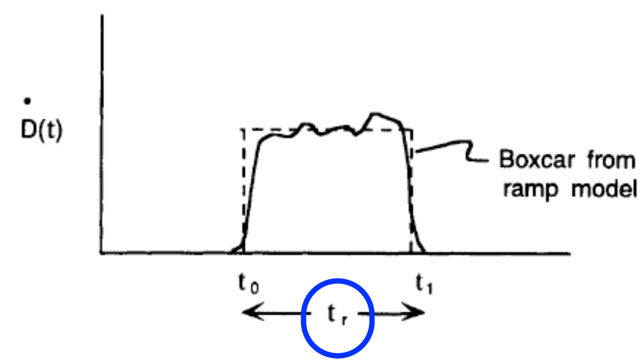
which means that the P-wave far-field displacement is proportional to the convolution of that Boxcar with the slip rate function of any individual subfault (i.e. recall that all subfault experience the same slip rate history).

Fault Model with Unidirectional Rupture Propagation

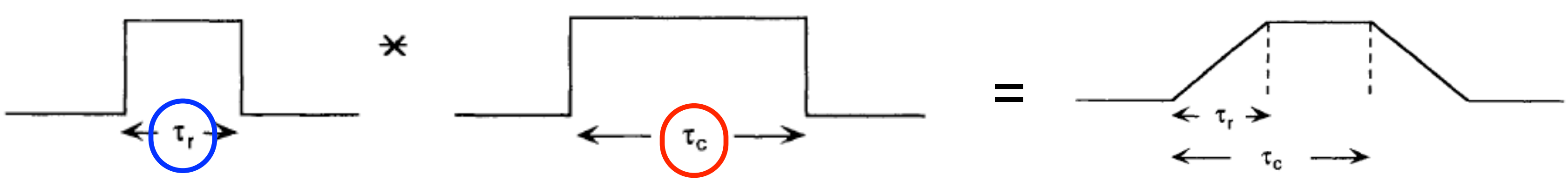
Assuming that the slip history has a constant rate until the final offset (i.e. a ramp function) then the slip rate $\dot{D}$ is a Boxcar function with rise-time t_r (figure).



$$u_r(r,t) = \frac{R^P \mu w}{4\pi\rho\alpha^3 r} v_r \dot{D}(t) * B(t;\tau_c).$$

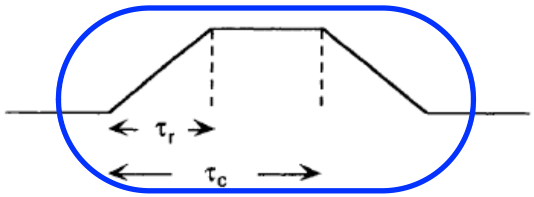


Because of the above equation, the P-wave far-field pulse shape is defined by the convolution of two boxcar functions, one representing the slip rate history of a single subfault, and the other representing the effect (duration) of the fault finiteness (i.e. $\tau_c = x/v_r$):

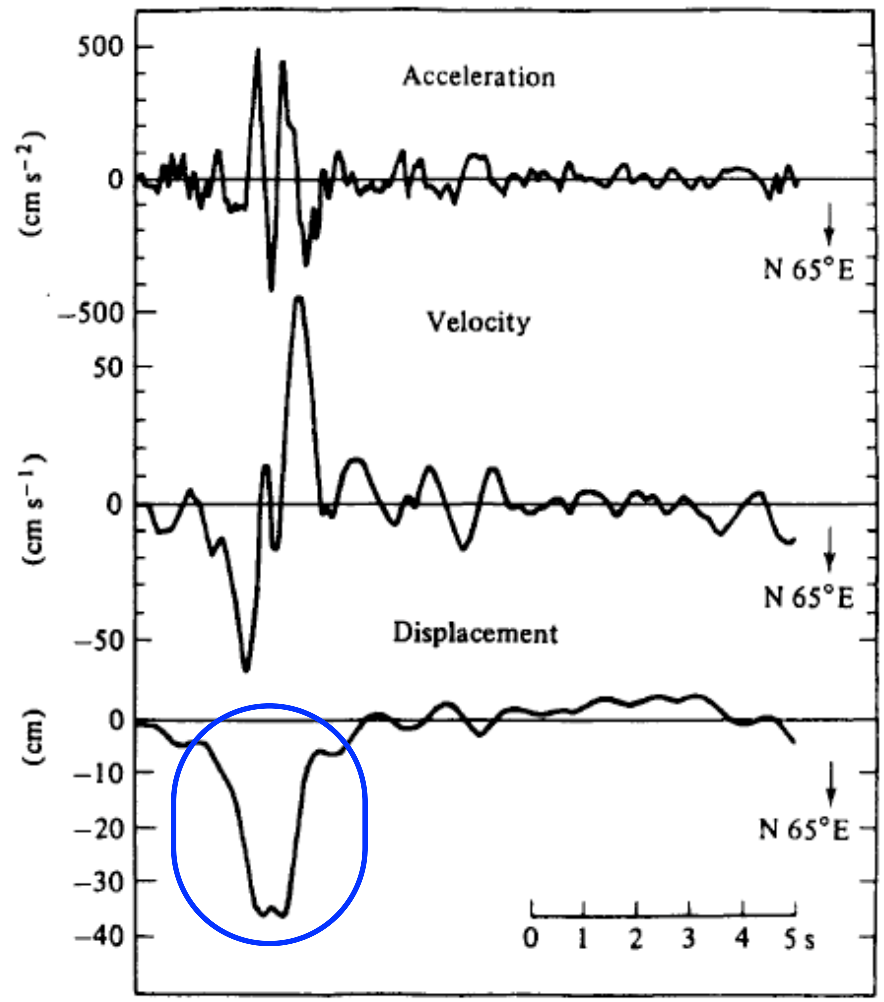


Fault Model with Unidirectional Rupture Propagation

The P or S-wave far-field displacement is defined as the apparent Source Time Function (STF):



Recordings of the ground motion near the epicenter of an earthquake at Parkfield, California (i.e. San Andreas fault). The station is located on a nodal for the P-wave and a maximum for the SH-wave. Notice the trapezoidal shape of the displacement pulse.



Parkfield earthquake (Aki, 1968)

Fault Model with Unidirectional Rupture Propagation

Time integrating the P (or S-wave) far-field displacement

$$\int_{-\infty}^{\infty} u_r(r,t)\,dt = \int_{-\infty}^{\infty} \frac{R^P \mu}{4\pi\rho\alpha^3} v_r \frac{w}{r} \dot{D}(t) * B(t;\tau_c)\,dt$$

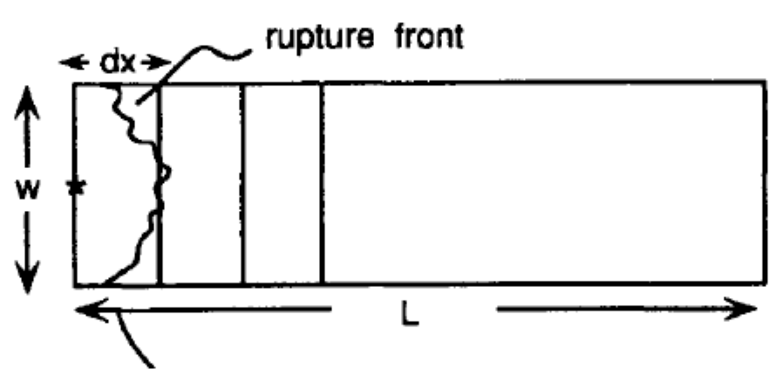
and since $\int_{-\infty}^{\infty} (f * g)\,dt = \left[\int_{-\infty}^{\infty} f(u)\,du\right]\left[\int_{-\infty}^{\infty} g(t)\,dt\right]$, arranging the terms we obtain

$$\frac{4\pi r\rho\alpha^3}{R^P} \int_{-\infty}^{\infty} u_r(r,t)\,dt = \int_{-\infty}^{\infty} \dot{D}(t)\,\mu w v_r B(t;\tau_c)\,dt.$$

The right-hand side represents the product of the average final slip D and the area of $\mu w v_r B(t;\tau_c)$, which is equal to μwL. Thus, since the product wL is the fault area A, we have that:

$$\int_{-\infty}^{\infty} u_r(r,t)\,dt \propto M_0 = \mu \bar{u} A$$

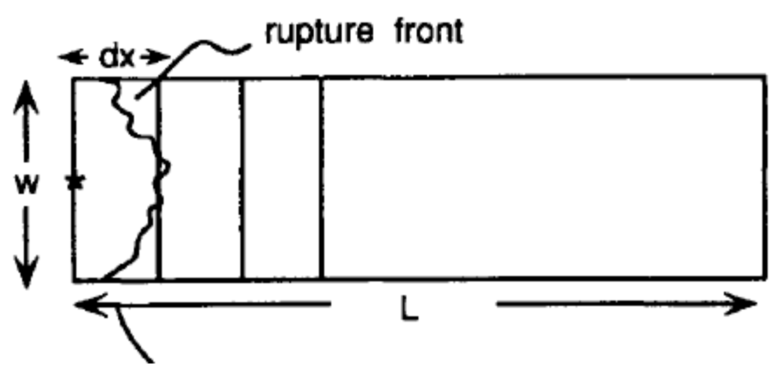
which is proportional to the seismic moment M_0.

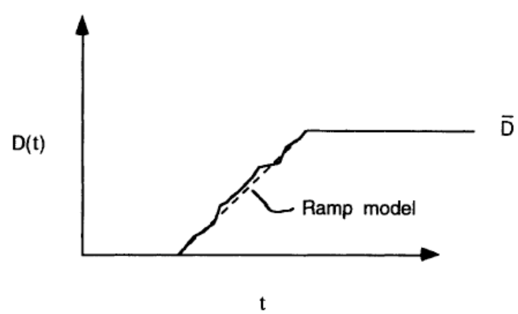


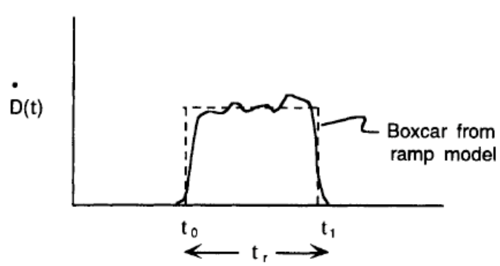
Fault Model with Unidirectional Rupture Propagation

The finite fault model we have introduced has been first studied by Haskell (BSSA, 1964 and 1969) and it is often called the Haskell's model. It depends on five basic source parameters:

1. Fault length (L)
2. Fault width (W)
3. Rupture velocity (v_r)
4. Final average slip (D)
5. Rise time (t_r)







For many earthquakes, reliable estimates of the product of L, W and D have been made, and hence of the seismic moment by assuming a value of rigidity.

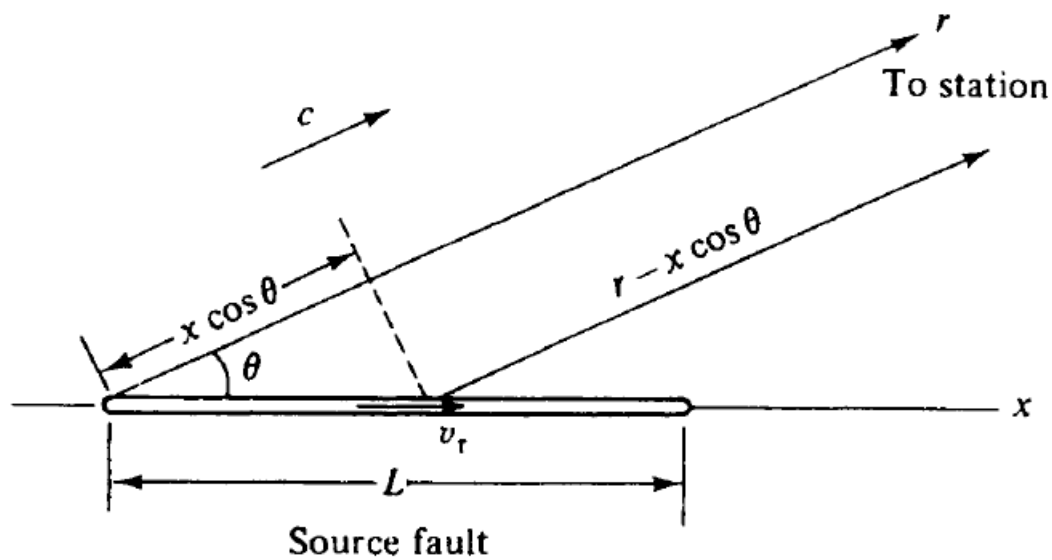
Reliable estimates of D and t_r require near-field data, which are difficult to obtain.

Source Directivity Effects

In the Haskell source model, the boxcar associated with rupture propagation has a length τ_c as seen in the far-field at a station located in the direction perpendicular to fault strike.

Such length obviously depends on both the fault dimensions and rupture velocity. However, it also depends on the azimuthal position of the station relative to the source (Doppler effect), as sketched in the figure below.

Body waves excited at the left extremity of the fault and traveling with speed c will propagate a longer distance than those excited at any position x later in the fault. Thus the arrival times of both waves packages are different.



Source Directivity Effects

The arrival time of a body-wave excited at the left fault extremity is t=(r/c), where r is the distance between the fault extremity and the receiver. The arrival time of a wave excited in a fault segment at point x is give by:

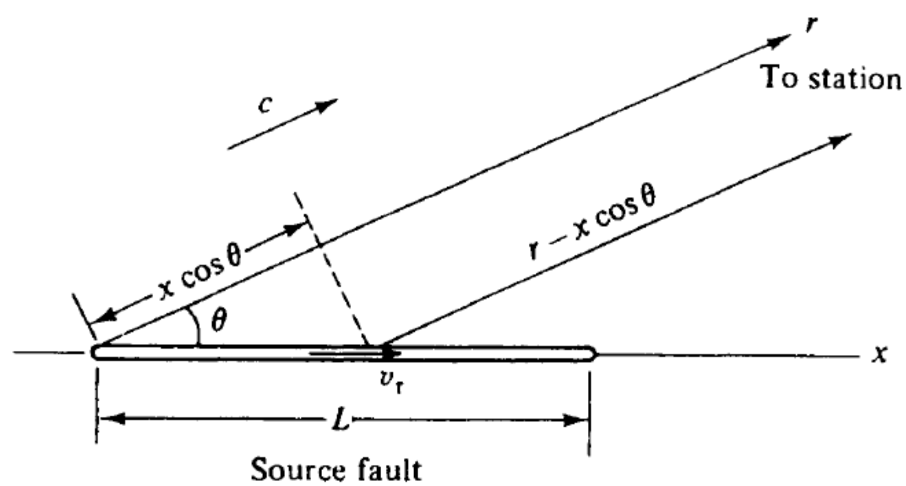
$$t_x = \frac{x}{v_r} + \frac{(r - x\cos\theta)}{c}.$$

The duration of the rupture propagation boxcar may thus be estimated as:

$$\tau_c = \left[\frac{L}{v_r} + \left(\frac{r - L\cos\theta}{c}\right)\right] - \left(\frac{r}{c}\right)$$

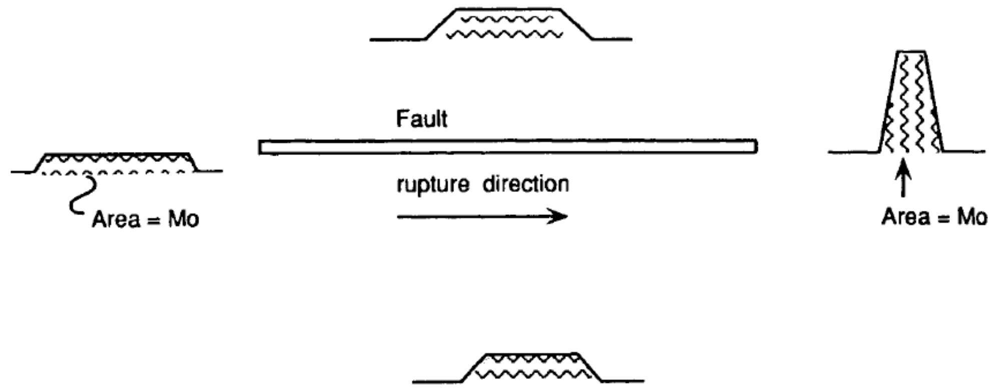
which reduces to a simple function of θ, the fault length and both rupture and wave speeds:

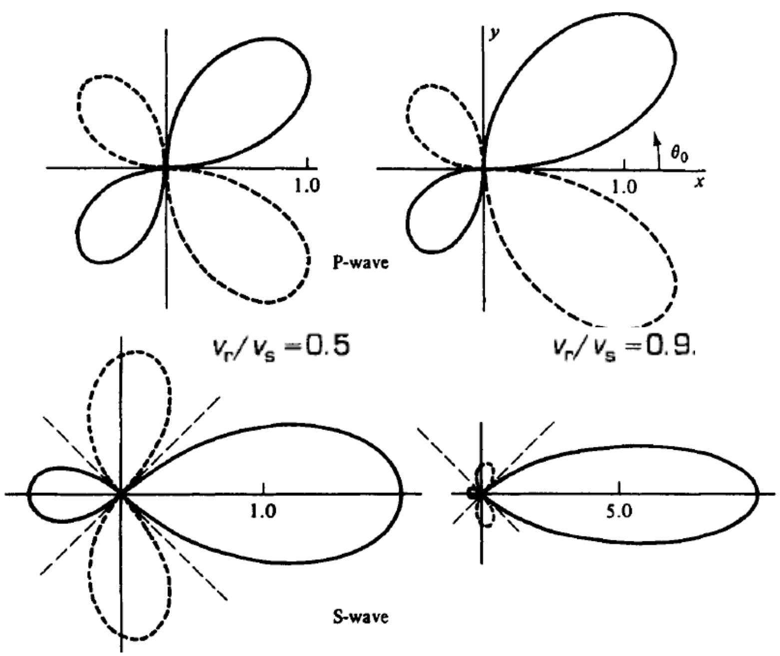
$$\tau_c = \frac{L}{v_r} - \left(\frac{L\cos\theta}{c}\right).$$



Source Directivity Effects

Since the duration of the apparent source time function (i.e. the body-wave far-field displacement) depends on the angle θ, and since the area under such function is proportional to the seismic moment M_0, then the amplitude of the STF also changes with the station position.





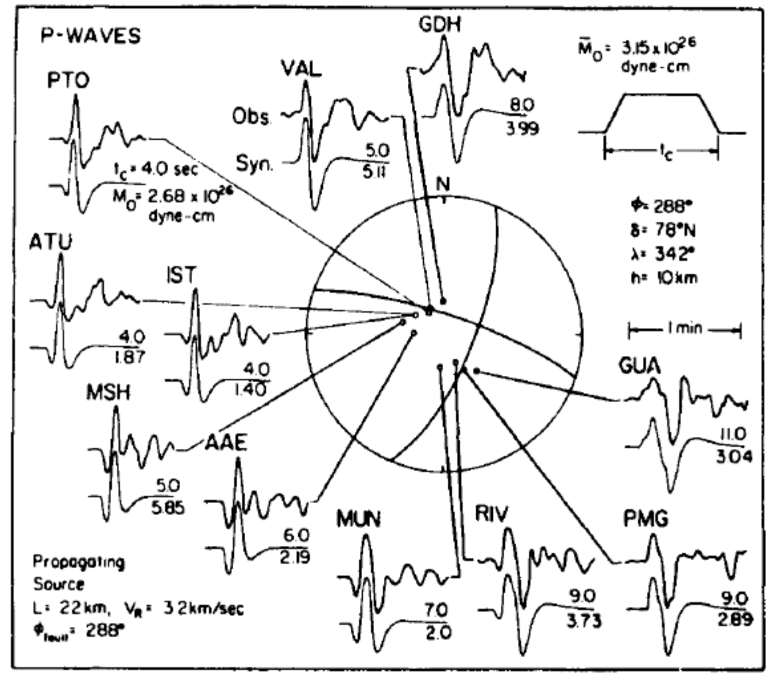
The apparent rupture duration τ_c also depends on the ratio of both the rupture and the wave speeds:

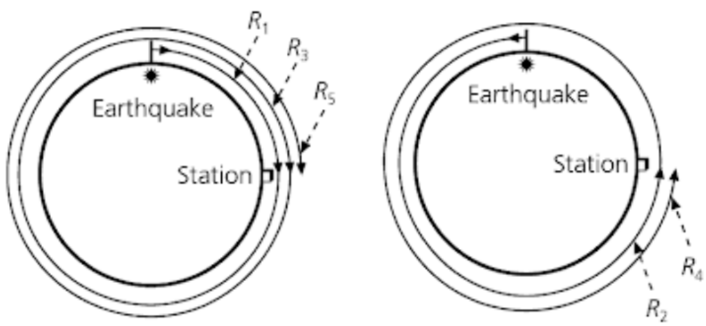
$$\tau_c = \frac{L}{v_r} - \left(\frac{L\cos\theta}{c}\right).$$

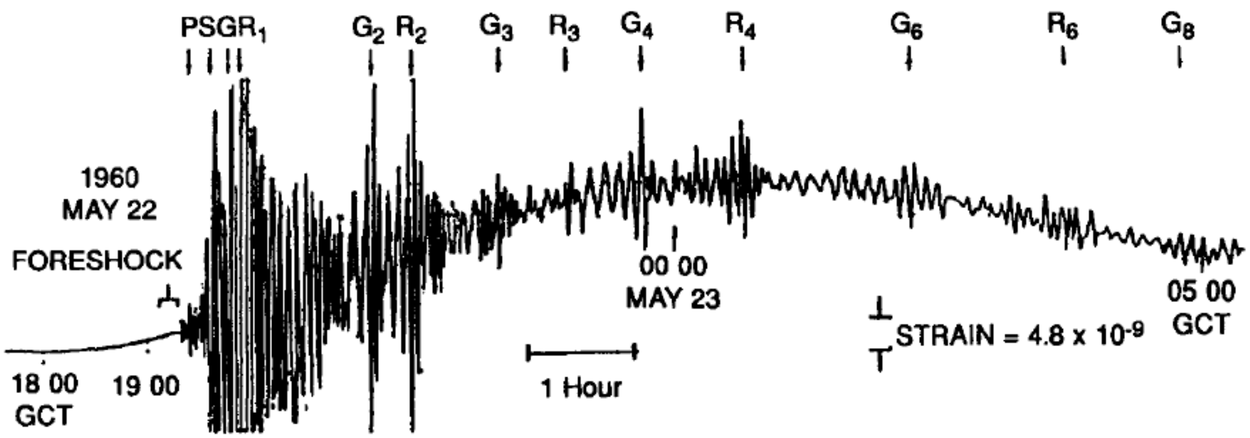
Then the radiation pattern for two different speed ratios changes significantly, specially for the S-waves (see figure).

Source Directivity Effects

Observed (above) and synthetic (below) P-wave seismograms for the 1975 Haicheng earthquake. Notice the strong directivity effect in stations located close to the along-strike direction (e.g. compare the pulse width in stations PTO and GUA)







Long-period record of the 1960 Chilean earthquake. Notice that the amplitude of the G4 and R4 arrivals is bigger than that of the G3 and R3 arrivals, which propagated a shorter distance, due to source directivity.