The Seismic Source Spectrum

1. Far-field finite source spectrum
2. Rupture directivity effects
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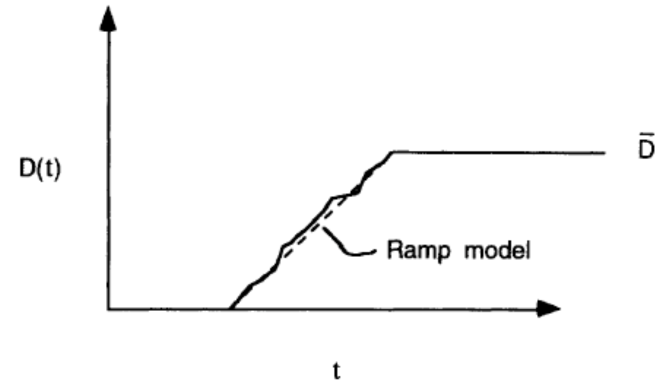
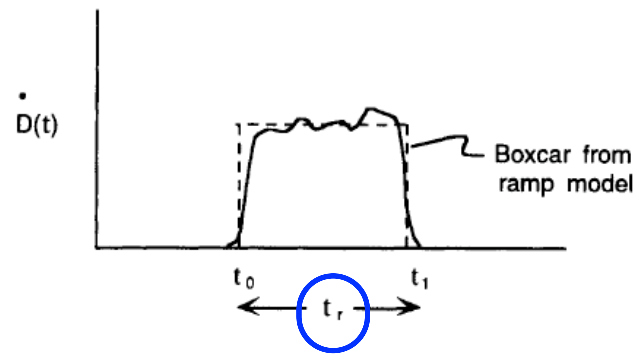
Far-Field Finite Source Spectrum

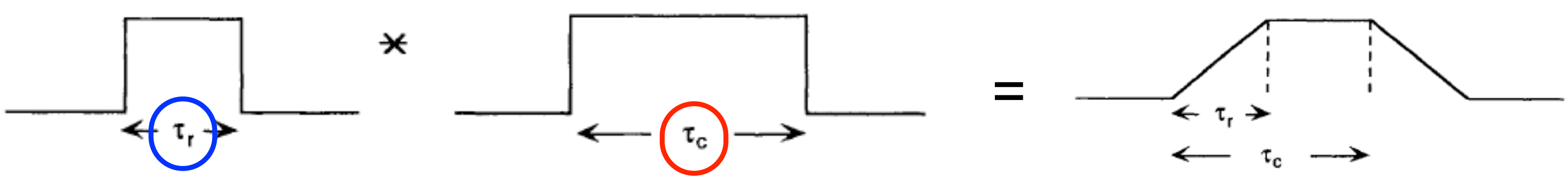
Consider a fault model with unidirectional rupture propagation (i.e. Haskell's model). The P-wave far-field displacement of such model is

$$u_r(r,t) = \frac{R^P \mu w}{4\pi\rho\alpha^3 r} v_r \dot{D}(t) * B(t;\tau_c).$$



Because of the above representation, the shape of the P-wave far-field pulse is defined by the convolution of two boxcar functions, one representing the slip velocity history of a single subfault, and the other representing the effect of the fault finiteness (i.e. $\tau_c = x/v_r$):





Far-Field Finite Source Spectrum

Time integrating the P (or S-wave) far-field displacement

$$\int_{-\infty}^{\infty} u_r(r,t)\,dt = \int_{-\infty}^{\infty} \frac{R^P \mu}{4\pi\rho\alpha^3} v_r \frac{w}{r} \dot{D}(t) * B(t;\tau_c)\,dt$$

leads to the seismic moment M_0:

$$\frac{4\pi r\rho\alpha^3}{R^P} \int_{-\infty}^{\infty} u_r(r,t)\,dt = \int_{-\infty}^{\infty} \dot{D}(t)\mu w v_r B(t;\tau_c)\,dt = M_0$$

To see this, notice that the right-hand side represents the product of the average final slip D and the area of $\mu w v_r B(t;\tau_c)$, which is equal to μwL with wL = A, the fault area.

We thus deduce that the displacement is equal to M_0 times the convolution of two boxcars (normalized by $1/\tau_r$ and $1/\tau_c$ respectively) with characteristic length durations:

$$u(t) = M_0\big(B(t;\tau_r) * B(t;\tau_c)\big).$$

Far-Field Finite Source Spectrum

The Fourier transform of a boxcar with height 1/T and length T is the Sinc function:

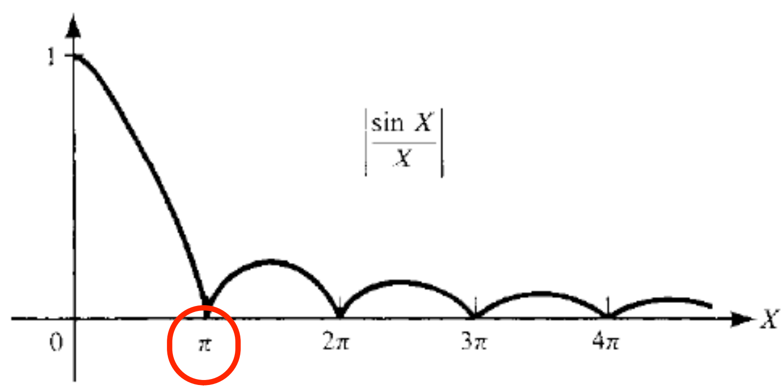
$$F(\omega) = \int_{-T/2}^{T/2} \frac{1}{T} e^{i\omega t} dt = \frac{1}{Ti\omega}\left(e^{i\omega T/2} - e^{-i\omega T/2}\right) = \frac{\sin(\omega T/2)}{\omega T/2}$$

and thus the spectral displacement amplitude is given by the product of the absolute value of two Sinc functions weighted by the seismic moment:

$$|A(\omega)| = M_0 \left| \frac{\sin(\omega T_R/2)}{\omega T_R/2} \right| \left| \frac{\sin(\omega T_D/2)}{\omega T_D/2} \right|$$

where ω is the frequency, and T_R and T_D are the apparent source duration (τ_c) and the rise time, respectively.

The Sinc function, which controls the shape of the far-field body wave spectrum, has nodes at X = π, 2π, …

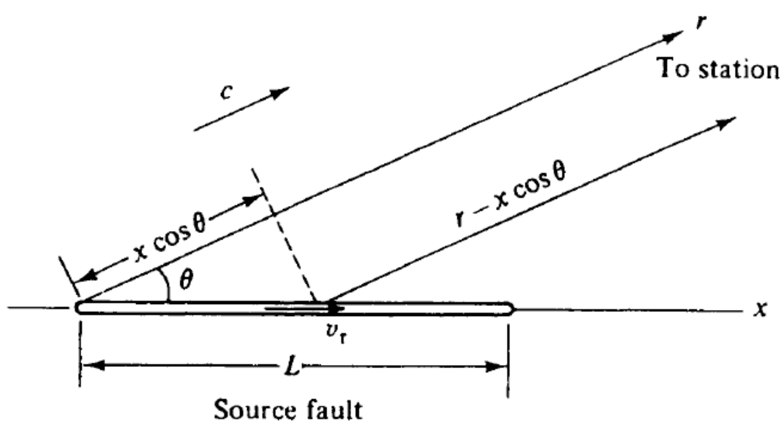


Far-Field Finite Source Spectrum

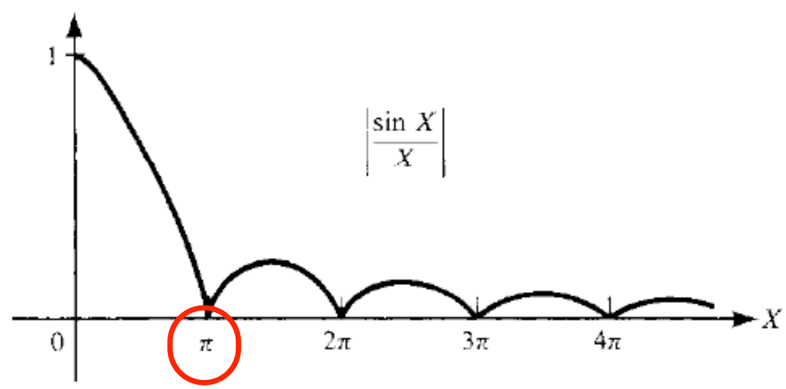
Since the effect of source finiteness is provided by a boxcar with length τ_c, the apparent rupture duration, throughout

$$u_r(r,t) = \frac{R^P \mu w}{4\pi\rho\alpha^3 r} v_r \dot{D}(t) * \boxed{B(t;\tau_c)}$$

and since τ_c is subject to the source directivity effect, which makes the displacement dependent on the source-receiver position, then the first spectrum node is due to the fault finiteness and corresponds to the period



$$2\pi/\omega = \tau_c = \boxed{\frac{L}{v_r} - \left(\frac{L\cos\theta}{c}\right)}$$



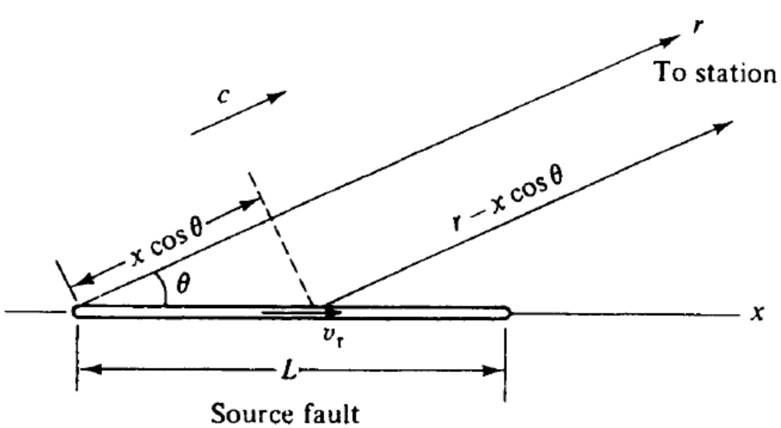
which contains information about the source length and rupture velocity.

Far-Field Finite Source Spectrum

Source directivity, which affects the displacement spectrum throughout factor

$$\left.\frac{\sin(\omega T_R/2)}{\omega T_R/2}\right| \quad \text{where} \quad T_R = \frac{L}{v_r} - \left(\frac{L\cos\theta}{c}\right).$$

induces a smoothing effect on the spectrum that is weakest in the direction of rupture propagation ($\theta=0$) and strongest in the opposite direction ($\theta=\pi$). As a result we observe more high-frequency waves when rupture approaches the receiver position.



The rise time has a smoothing effect in the source spectrum as well but, since in general $T_D < T_R$, such effect affects higher frequencies than the source finiteness.

$$\frac{\sin(\omega T_D/2)}{\omega T_D/2}$$

Far-Field Finite Source Spectrum

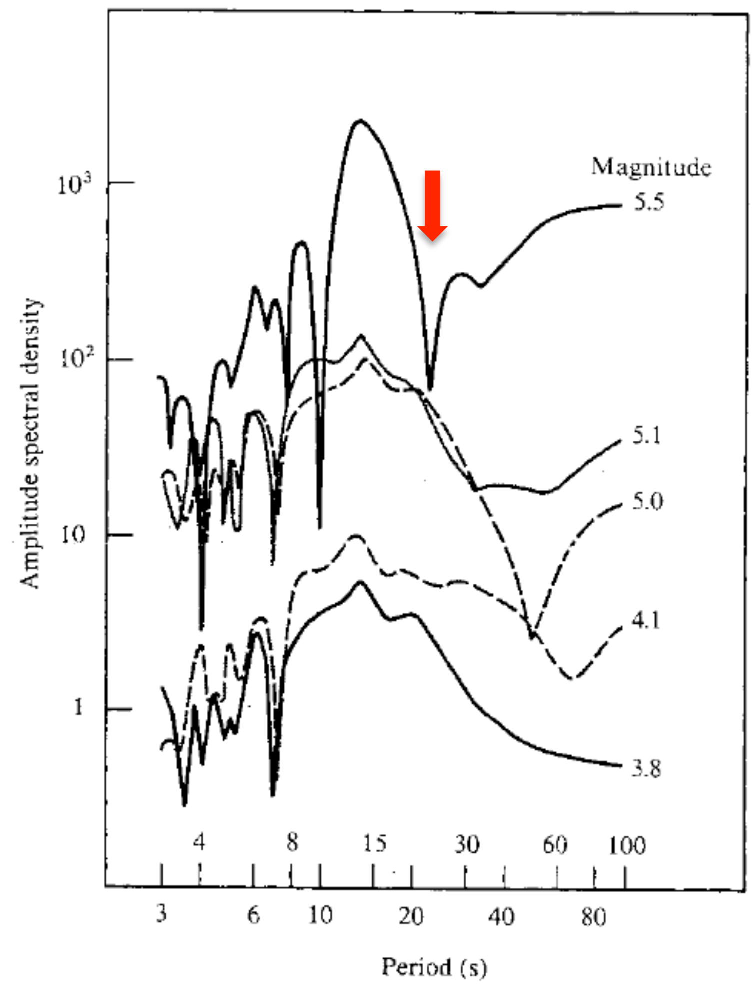
Amplitude spectra of Love waves from a series of earthuakes in Parkfield (June 1966), California, recorded at Berkeley, at a distance of 270 km.

$$\frac{\sin(\omega T_R/2)}{\omega T_R/2}$$

The far-field displacement spectrum decays as $1/\omega$

The first node at period T=22.5 s is explained by the rupture velocity of 2.2 km/s, the fault length of about 20 km and cos(θ) of about 1:

$$T_R = \frac{L}{v_r} - \left(\frac{L\cos\theta}{c}\right) = \frac{2\pi}{\omega}$$

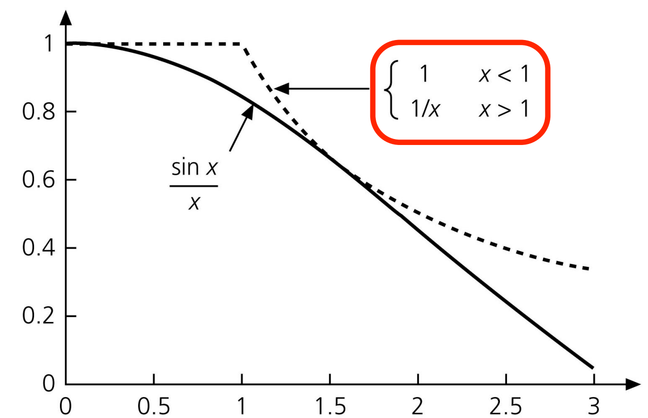


Far-Field Finite Source Spectrum

The logarithm of source spectrum is given by the sum

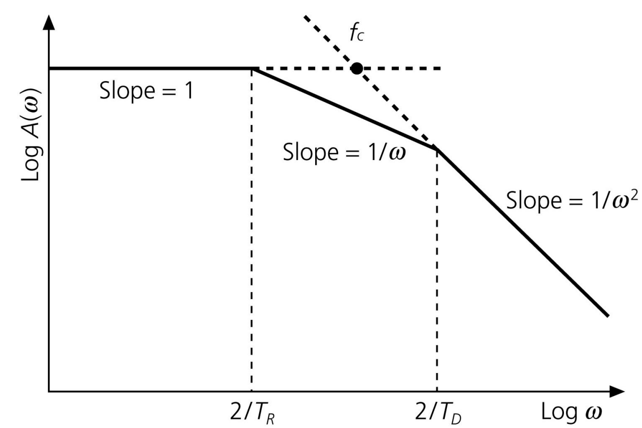
$$\log\ A(\omega) = \log\ M_0 + \log\left[\operatorname{sinc}(\omega T_R/2)\right] + \log\left[\operatorname{sinc}(\omega T_D/2)\right]$$

By approximating the Sinc functions as shown in the figure (top), the equation above represents an approximation of the amplitude spectrum of a trapezoidal boxcar function defined as:



$$\log|A(\omega)| =$$

$$= \left\{\begin{array}{ll} \boxed{\log\ M_0} & \omega < 2/T_R \\ \log\ M_0 - \log\,(T_R/2) - \log\omega & 2/T_R < \omega < 2/T_D \\ \log\ M_0 - \log\,(T_R T_D/4) - 2\log\omega & 2/T_D < \omega \end{array}\right\}$$



This theoretical source spectrum has three regions with slopes 1, ω^{-1} and ω^{-2} separated by two corner frequencies which depend on both the rise time (T_D) and the rupture duration (T_R).

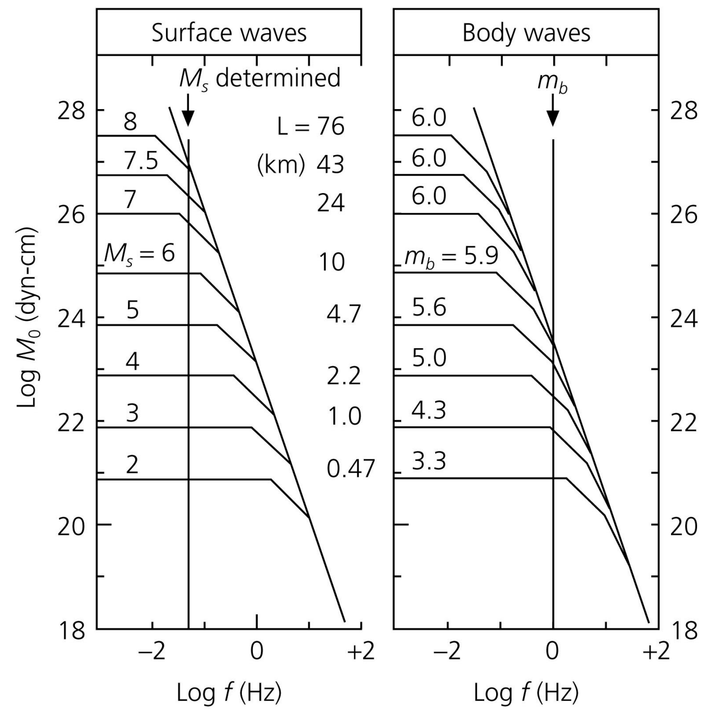
Far-Field Finite Source Spectrum

The displacement source spectrum is flat and proportional to M_0 at low frequencies. For high frequencies it decays as ω^{-2}. Thus, the spectrum is parameterized by three factors:

1. Seismic moment,
2. Rise time and
3. Rupture duration.

There exist other source models with different spectra, such as the one considering rupture propagation in the along-width direction as well, which introduces an even faster decay in high frequencies (as ω^{-3}).

Theoretical spectra for different earthquakes' magnitudes. Notice the spectrum saturation in both body and surface waves magnitudes at periods of 1 and 20 s, respectively.

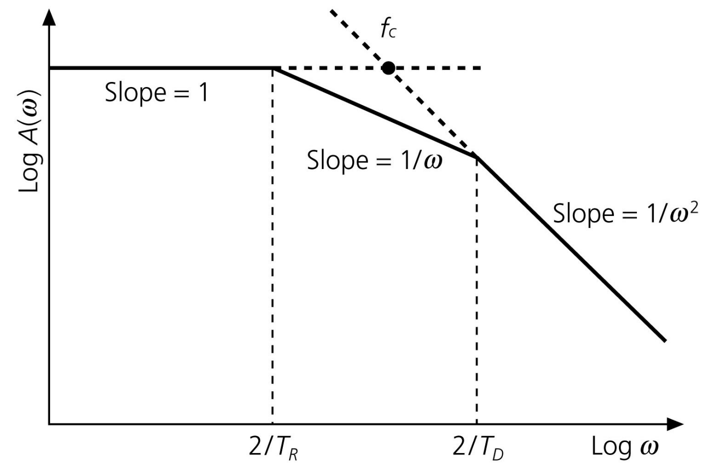


Far-Field Source Spectrum

Following Brune (JGR, 1970) for a point dislocation, the far-field displacement spectrum is roughly characterized by three parameters:

1. The low-frequency level, which is proportional to M_0
2. The corner frequency f_c; and
3. The power of the high-frequency asymptote

Is it possible to extract fundamental source information from the spectrum of the far-field displacement of body-waves?



Assuming bilateral faulting with rupture velocity v and final length L, Savage (1972) calculated the P and S-waves corner frequencies from the slip function:

$$\begin{aligned}\Delta u(\xi, t) &= D_0 G(t - \xi_1/v) && 0 \le \xi_1 < L/2 \\ &= D_0 G(t + \xi_1/v) && -L/2 < \xi_1 < 0 \\ &= 0 && \text{otherwise,}\end{aligned}$$

where

$$\begin{aligned}G(t) &= 0 && t < 0 \\ &= 1 - \exp(-t/T) && 0 \le t.\end{aligned}$$

Far-Field Source Spectrum

Assuming that rise time T is equal to the travel time of the rupture front over half a fault width, Savage (1972) obtained single corner frequencies f_c per body-wave as a geometric mean of the corner frequencies associated with both the rupture time and the rise time, so that

$$2\pi \langle f_P \rangle = \sqrt{2.9} \cdot \alpha / \sqrt{LW} \qquad 2\pi \langle f_S \rangle = \sqrt{14.8} \cdot \beta / \sqrt{LW}$$

In this case, the high-frequency asymptote is proportional to ω^{-2}. Thus, from observed corner frequencies, the source area (*LW*) may be estimated. However, contradicting most of observations, this model always predicts $f_P < f_S$.

Sato and Hirasawa (1973) introduced a source spectrum based on a circular-crack model propagating radially with constant velocity that also exhibits an asymptote like ω^{-2} but predicts $f_P > f_S$. Their corner frequencies over all directions for the P and S waves are

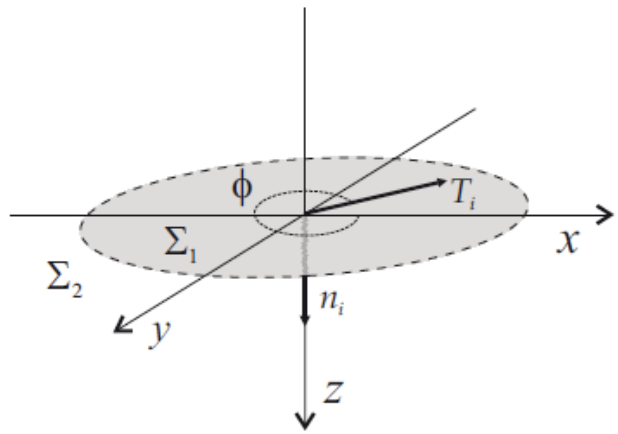
$$2\pi \langle f_P \rangle = C_P \alpha / R \qquad 2\pi \langle f_S \rangle = C_S \beta / R$$

where R is the source radius and C_p and C_s are functions of rupture velocity.

Far-Field Source Spectrum

James Brune (1970) introduced a source model whose spectrum is parameterized in terms of the source stress drop and fault dimensions.

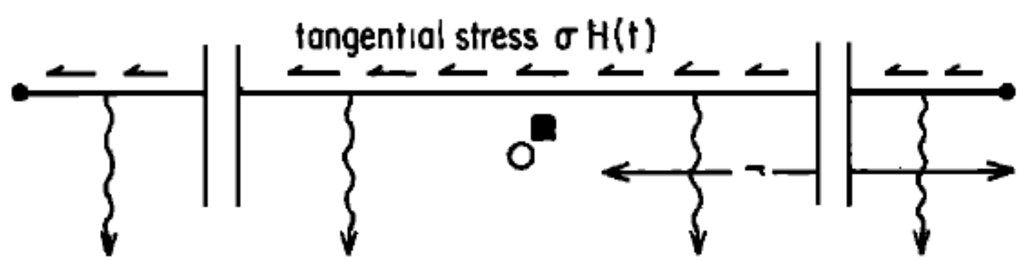
Brune's model assumes a shear dislocation in a circular crack due to a sudden stress drop (σ or $\Delta\sigma$) simultaneously throughout the entire crack.



The stress wave propagating from the center of the crack up to a receiver close to it is

$$\sigma(x,\, t) = \sigma H(t - x/\beta)$$

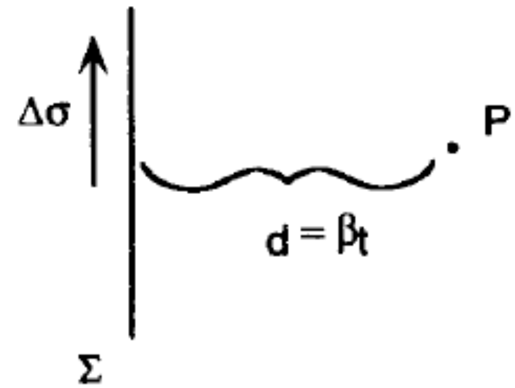
where H(t) is the Heaviside function. From the Lagrangian strain tensor definition and Hooke's law, such stress drop is given by $\sigma = \mu\partial u/\partial x,$



Far-Field Source Spectrum

The strain associated with the stress perturbation $\sigma = \mu \partial u / \partial x$, may be approximated as u/(βt) so that the fault-parallel displacement at the receiver close to the crack is

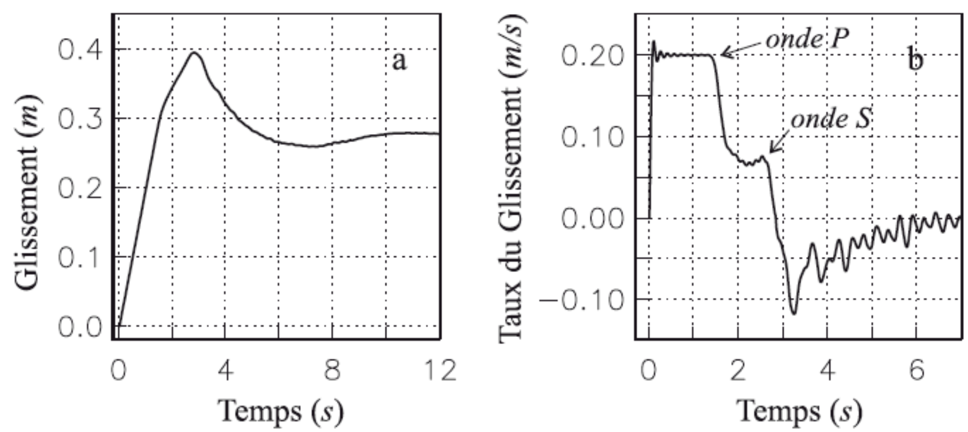
$$u(t) = \frac{\Delta\sigma\beta t}{\mu}.$$



thus the particle velocity is directly proportional to both the stress drop and S-wave speed, and inversely proportional to rigidity

$$\dot{u}(t) = \Delta\sigma(\beta/\mu)$$

Assuming a Poissonian solid with β = 1 km/s and Δσ = 0.1 MPa, both the particle displa-cement and velocity are (Cruz-Atienza, PhD Thesis, 2006):



Far-Field Source Spectrum

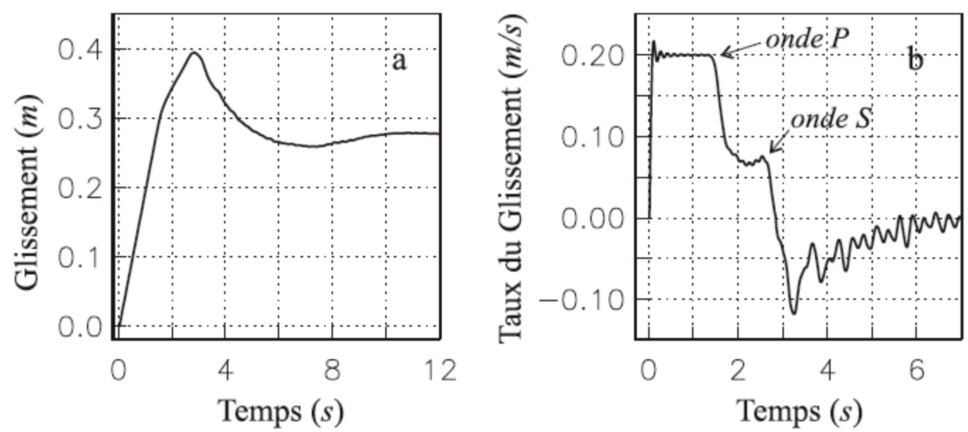
The static (i.e. final) slip for this problem has a closed form found by Eshelby (1957) given by

$$S(x,y) = \frac{24}{7\pi}\frac{\Delta\tau}{\mu}\left[r^2 - \left(x^2+y^2\right)\right]^{\frac{1}{2}}.$$

By inputting the model parameters into such formula we obtain a final slip at the fault center equal to 0.27 m, which coincides with the numerical prediction shown in the figure below for t = 12 s.

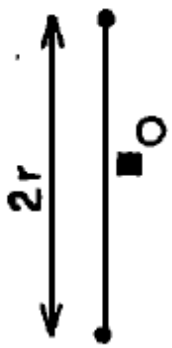
Notice that particle velocity is constant and equal to 0.2 m/s until the arrival of diffracted waves (P and S-stopping phases) generated at the crack edge.

Assuming a Poissonian solid with β = 1 km/s and $\Delta\sigma$ = 0.1 MPa, both the particle displacement and velocity are (Cruz-Atienza, PhD Thesis, 2006):



Far-Field Source Spectrum

To approximate the near-field displacement considering the diffracted waves, Brune multiplied the displacement function by an exponential with characteristic time, τ, equal to r/β, where r is the fault radius.

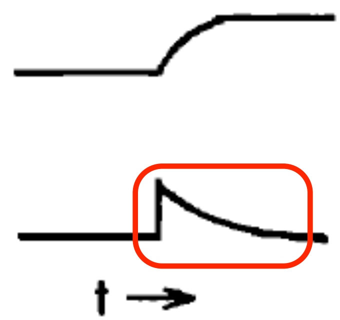


at point ○

$$u = \frac{\sigma}{\mu}\beta\tau\left(1-e^{-t/\tau}\right)$$

$$\dot{u} = \frac{\sigma}{\mu}\beta e^{-t/\tau}$$

$$\tau \sim \frac{r}{\beta}$$



Notice that the velocity initial rise is not affected with respect to that of an infinite source but decays to zero with time.

Far-Field Source Spectrum

In the far-field, once corrected the source radiation pattern, and considering an amplitude spherical spreading with distance R from the source, the spectrum of the displacement becomes

$$\langle \Omega(\omega) \rangle = \langle \mathcal{R}_{\theta\phi} \rangle \frac{\sigma\beta}{\mu} \frac{r}{R} F(\epsilon) \frac{1}{\omega^2 + \alpha^2}$$

where function $F(\varepsilon)$ has been chosen so that the spectrum in the long-period limit agrees with the source scalar moment and in the high-frequency limit conserves the energy-density flux at large distances:

$$F(\epsilon) = \{[2 - 2\epsilon] \\ [1 - \cos(1.21\, \epsilon\omega/\alpha)] + \epsilon^2\}^{1/2}$$

Thus, the Brune's source model depends on three basic parameters:

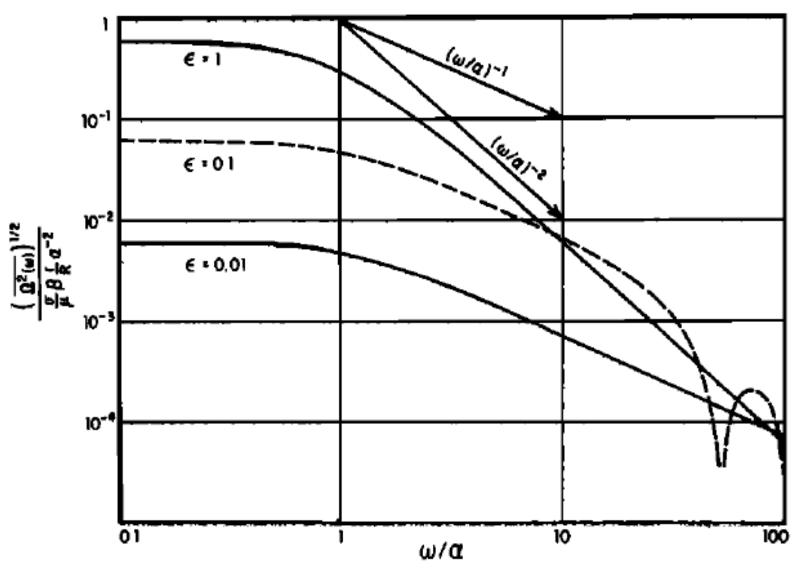
1. The effective stress drop (σ)
2. The source characteristic length (r); and
3. The corner frequency (α)

Far-Field Source Spectrum

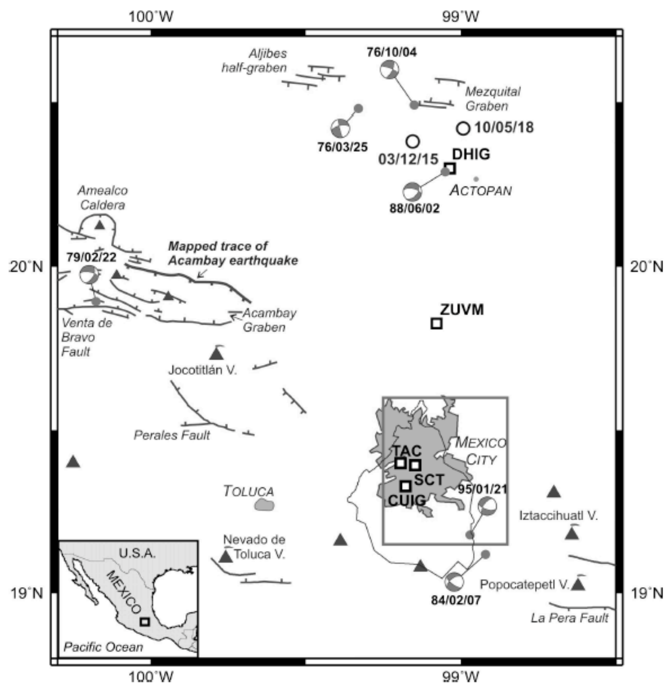
By fitting the observed spectra with the theoretical source spectrum predicted by Brune's model, we may solve for both the effective stress drop σ and source dimension r.

$$\langle \Omega(\omega) \rangle = \langle \mathcal{R}_{\theta\phi} \rangle \frac{\sigma\beta}{\mu} \frac{r}{R} F(\epsilon) \frac{1}{\omega^2 + \alpha^2}$$

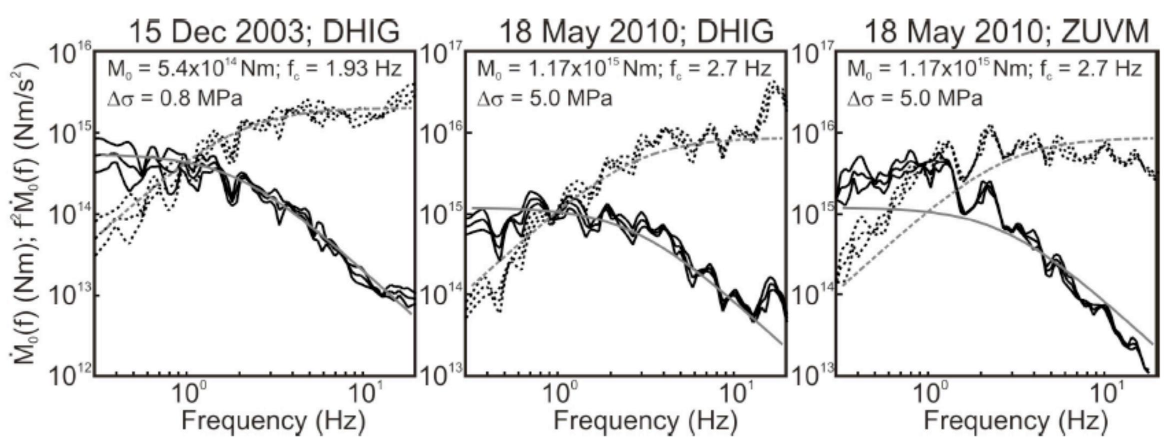
This source model, at low-frequencies approaches the seismic moment M_0. For values of ω/α near 1 the spectrum begins to fall as ω^{-1}, and for high values of ω/α the spectrum decays as ω^{-2}, which agrees with the source model introduced by Aki (1968)



Far-Field Source Spectrum

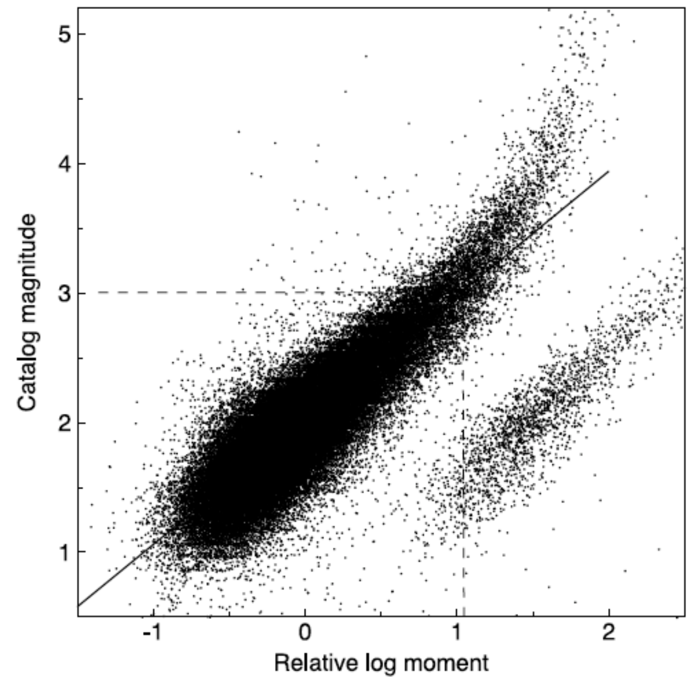


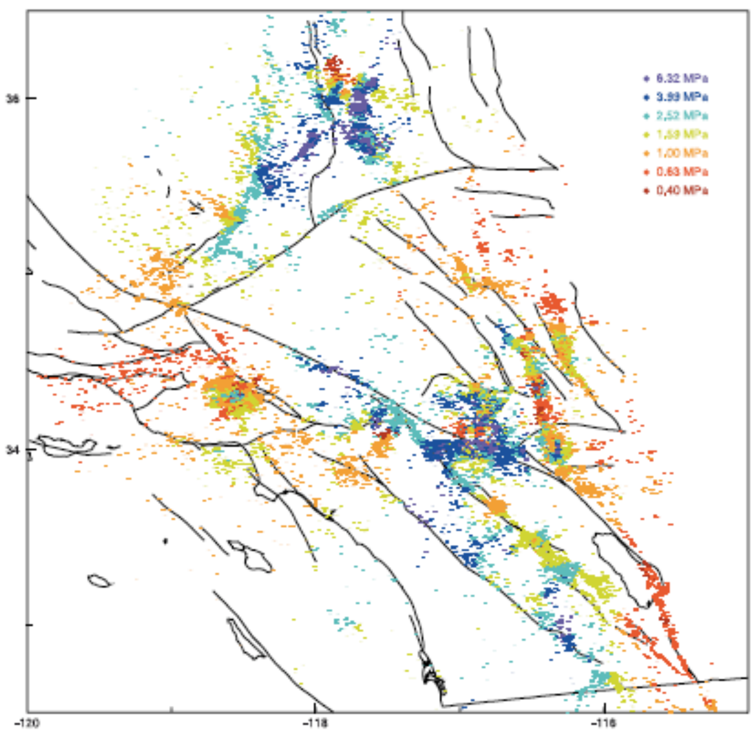
Brune's source spectrum model may be used to estimate seismic moment and stress drop for regional earthquakes after correcting seismograms for regional intrinsic attenuation (Q), like in this case for three events occurred in the MVB to the north of Mexico City (Singh et al., 2011).



Far-Field Source Spectrum

Or even apply systematically Brune's model for stress drop determination of thousands of events taking care of regional attenuation that also affects the spectrum, like in this example in southern California (Shearer et al., 2006)





Stress release may thus be studied regionally and associated to fault systems in the Earth's crust.