Dynamic Fracture Mechanics

1. Fault slip instability
2. Stick-slip fault model
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4. Elastic fields in a brittle fracture
5. Cohesive forces and slip-weakening
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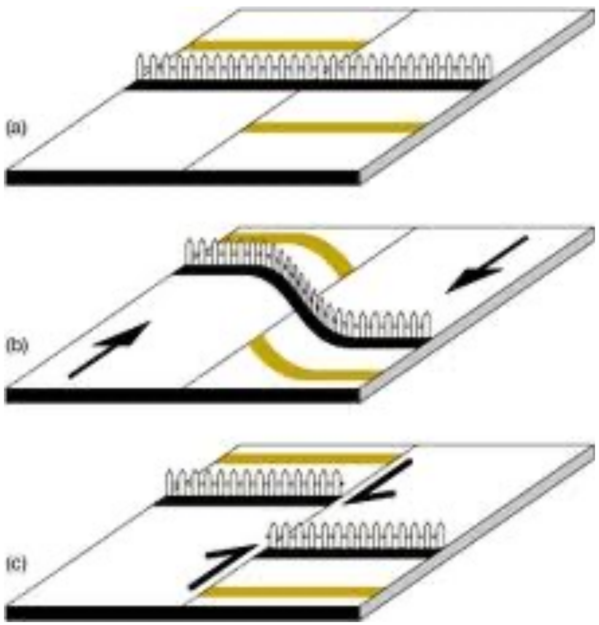
Macroscopic Fault Mechanics

From observations of the 1906 San Francisco earthquake, Henry F. Reid postulated the elastic rebound theory as the mean to produce earthquakes. An earthquake is the sudden release of previously stored elastic stress (i.e. strain).



Coulomb (1773) introduced a simple theory for rocks failure, so that the rock strength (τ) depends on three parameters: 1) cohesion, c, 2) coefficient of internal friction, μ_i, and 3) the normal traction, σ_n, on the plane of failure:

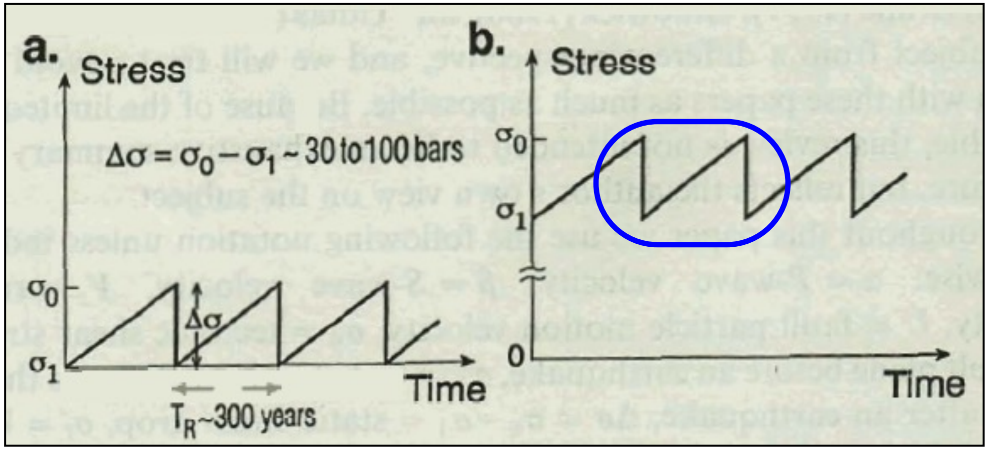
$$|\tau|_{\text{failure}} = c + \mu_i \sigma_n$$

If rupture happens in a preexisting fault, the fault strength essentially depends on the normal traction and μ_s, the coefficient of friction, which doesn't have the same value μ_i required for failure on fresh rocks. We thus have the following relationship, known as Amonton's second law (1699):

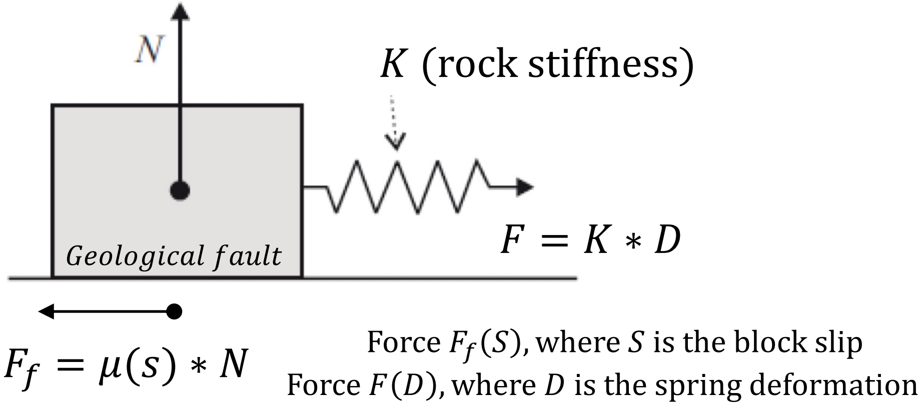
$$\tau = \mu_s \sigma$$

Macroscopic Fault Mechanics

Fault slip is always accompanied by a stress drop. Once the elastic energy has been released, strain starts cumulating again and a new seismic cycle begins. This unstable frictional behavior is known as “stick-slip”, concept introduced by Brace and Byerlee (1966).



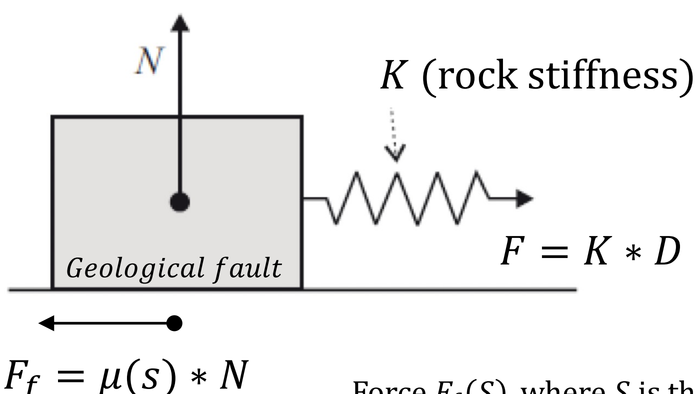
The elastic rebound happens when conditions for unstable sliding are set in the fault plane. These conditions are schematically shown in the figure, where force F represents the fault traction, F_f the fault friction and the stiffness of the spring, K, represents the elastic modulus of the medium surrounding the fault.



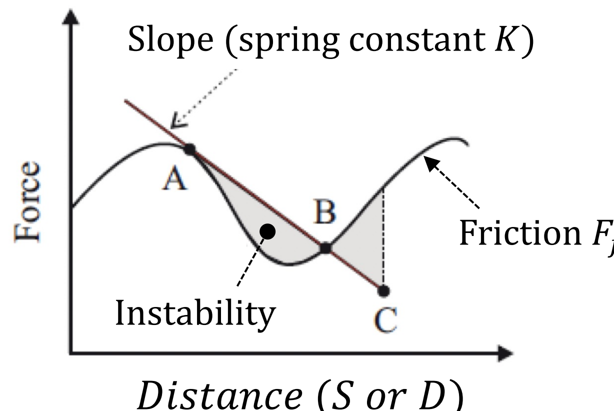
The slip instability (i.e. the rupture) happens if the weakening rate of the fault friction, F_f, is larger than the weakening rate of the fault shear traction, which is proportional to K.

Stability Involves Balance and Calm

- Potentially unstable systems remain stable (inertial or static) as long as certain conditions prevail.
- In mechanics, such conditions imply the balance of all acting forces during the evolution of the system.



Force $F_f(S)$, where S is the block slip
Force $F(D)$, where D is the spring deformation



Instability condition

$$\left|\frac{\partial F_f}{\partial S}\right| > K$$

This condition implies an imbalance of forces and thus acceleration

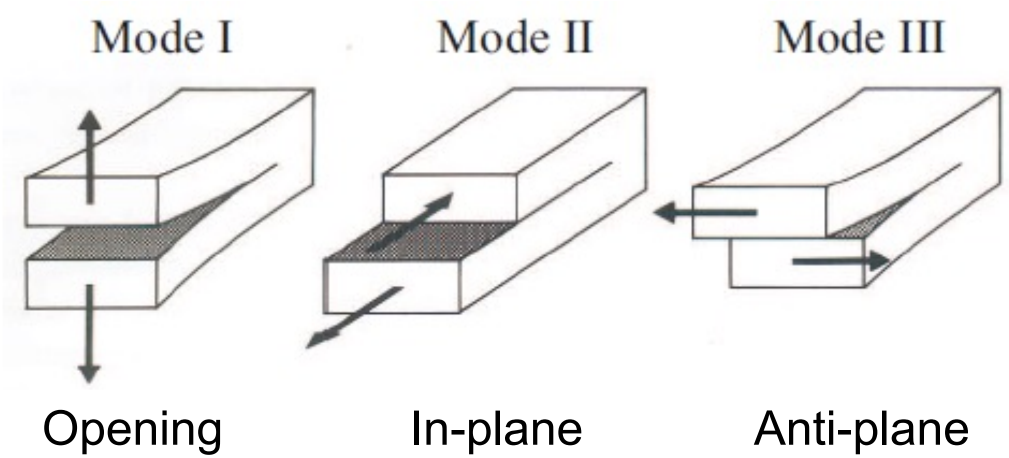
Macroscopic Fault Mechanics

From the slip instability condition we see that the fault strength (friction) evolution during an earthquake is critical to either promote or prevent rupture propagation.

Amonton's second law tells us that the fault strength is proportional to μ, the coefficient of friction. Assuming constant the fault normal traction, then rupture only depends on the time evolution of μ (i.e. the fault friction coefficient).

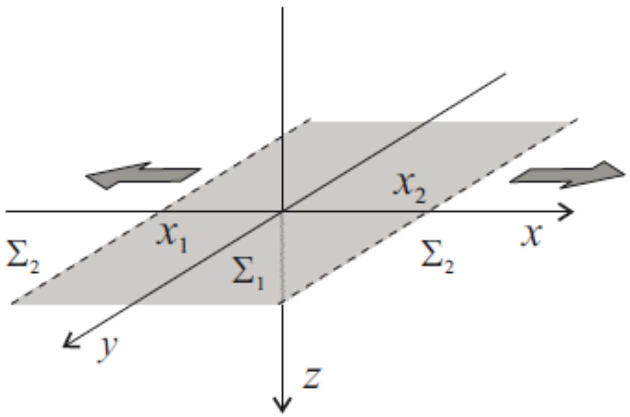
The Fault friction coefficient is a function of many different physical processes taking place during rupture, as temperature, slip rate, slip history and some state variables varying with time.

Let us first study the elastic fields produced by a faulting process in the vicinity of the rupture front. Any faulting mechanism may be thought as a linear combination of three fundamental modes of deformation:



Microscopic Fault Dynamics

Assume a given displacement discontinuity across Σ_1. Regardless of the deformation mode, the stress tensor over Σ_2 in the vicinity of the crack tip has a universal form given by:



$$\tau_{ij} = \frac{K(t)}{\sqrt{2\pi x'}} \cdot \Theta_{ij}(\theta)$$

where $x' = x - v_r t$ and v_r is the rupture front velocity. In this expression, both functions K(t) and $\Theta_{ij}(\theta)$ are scalars and depend on the rupture mode. The first one is know as the stress intensity factor and has units of Nw/m$^{3/2}$. The second one is a non-dimensional spatial pattern that depends on the position over the plane perpendicular to the crack tip (i.e. y=0 and angle θ). It ranges between zero and one, so that $\Theta_{ij}(0)$=1 over the plane $z = 0$ (Freund, 1990).

Notice the stress singularity at the crack tip (i.e. $x' = 0$), which has the form $1/\sqrt{x'}$

Microscopic Fault Dynamics

Now consider a slip function S that is 0 for x' > 0, and given by the logarithm of $|-x'|$ as show in the figure for x' < 0. The shear stress outside the crack is (Aki and Richards, 2002):

$$\tau_{iz} = \Phi \int_{-\infty}^{0} \frac{\dot{S}_i}{\xi - x'} d\xi,$$

where Φ is a constant that depends on μ, v_r, and α (if $i = x$, *in-plane*) or β (if $i = y$, *anti-plane*). This means that the stress field is equal to the Hilbert transform of the slip rate, $\dot{S}_i$, times a constant. Thus both fields have the same shape but with phase shifted of $\pi/2$.

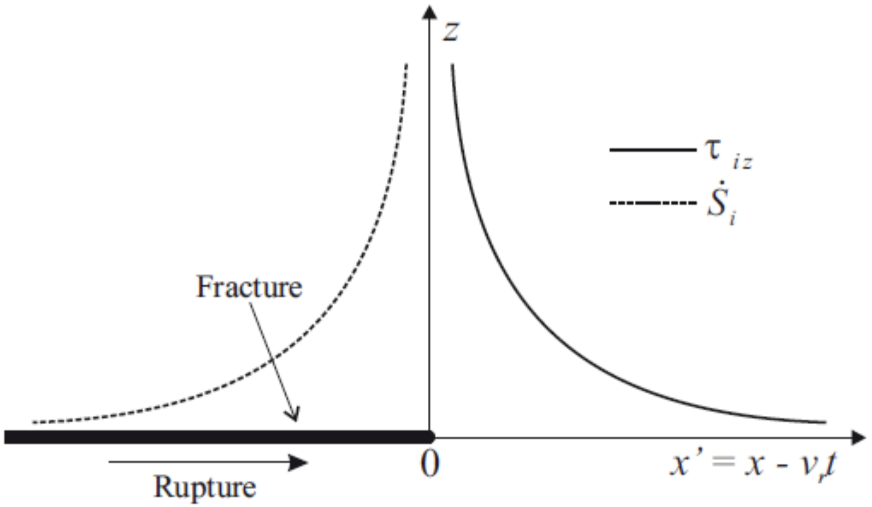
Stress and slip rate functions satisfying this equation in the z = 0 plane are

$$\tau_{iz} = \frac{K^m}{\sqrt{2\pi x'}} H(x') \qquad \dot{S}_i = \frac{A^m v_r}{2\sqrt{-x'}} H(-x').$$

where functions K^m and A^m are, respectively, the stress intensity factor and a constant, both depending on the rupture mode m (Aki and Richards, 2002, eqs. 11.6 and 11.7)

Microscopic Fault Dynamics

Therefore, the slip rate and shear stress have the form shown in the figure, both of them with a singularity at the crack tip.



We thus have that the stress and slip rate functions for a moving crack with spatial logarithmic slip distribution are

$$\tau_{iz} = \frac{K^m}{\sqrt{2\pi x'}} H(x') \qquad \dot{S}_i = \frac{A^m v_r}{2\sqrt{-x'}} H(-x'),$$

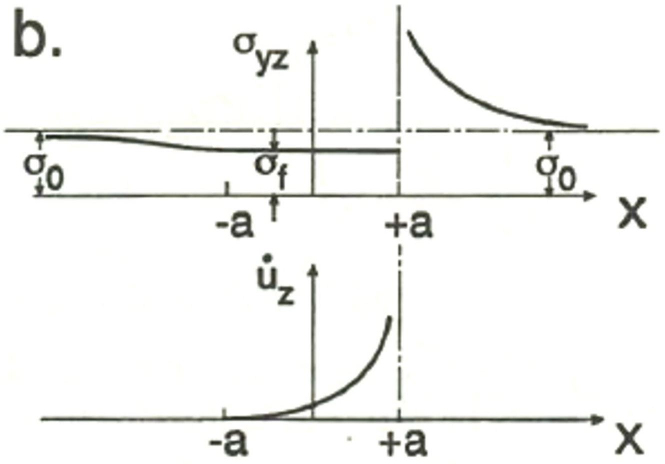
where functions K^m and A^m are, respectively, the stress intensity factor and a constant, both depending on the rupture mode m.

Unidirectional Anti-Plane Moving Crack

Consider now a unilateral anti-plane rupture (mode III) propagation with velocity v_r and fault length of 2a. The fault is loaded before rupture with a homogeneous shear stress τ_0. During propagation, an instantaneous and constant stress drop $\Delta\tau = \tau_0 - \tau_s$ takes place in the crack tip, where τ_s is the dynamic stress (Freund, 1979). Thus, the shear stress ahead the crack tip and the slip rate within the crack are respectively given by (Kanamori, 1994):

$$\tau_{yz} = \Delta\tau\left[\sqrt{\frac{x+a}{x-a}} - 1\right] + \tau_0$$

$$v_y = \Delta\tau\frac{v_r}{\mu}\sqrt{\frac{a+x}{a-x}}$$



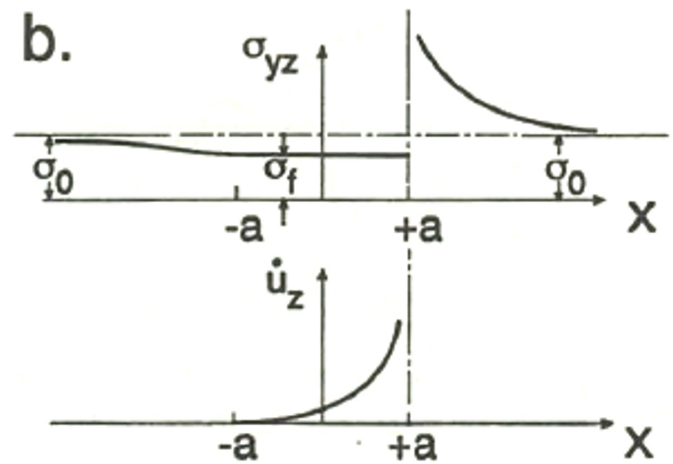
Since seismological observations give access to average source information, by integrating the slip rate over the fault we get particle velocity and stress drop:

$$\|v\| = \frac{1}{2a}\int_{-a}^{+a} v_y dx = \frac{\pi v_r}{2\mu}\Delta\tau$$

$$\Delta\tau = \frac{2\mu}{\pi v_r}\|v\| \approx \frac{\mu}{\beta}\|v\|$$

Unidirectional Anti-Plane Moving Crack

As far as the finite fault effects don't play a relevant role (e.g. diffracted stopping phases), particle velocity in the near field and stress drop in a moving crack with constant velocity are in accordance with prediction by Brune (1970) for his penny-shape instantaneous source model by a factor of 0.5, which is within uncertainties in seismological interpretation.

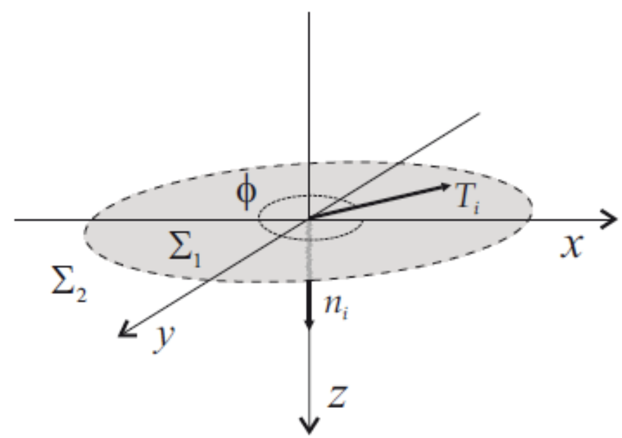


Penny-shape Brune's model

$$\|v\| = \frac{1}{2a}\int_{-a}^{+a} v_y dx = \frac{\pi v_r}{2\mu}\Delta\tau$$

$$\Delta\tau = \frac{2\mu}{\pi v_r}\|v\| \approx \frac{\mu}{\beta}\|v\|$$

Brune's model



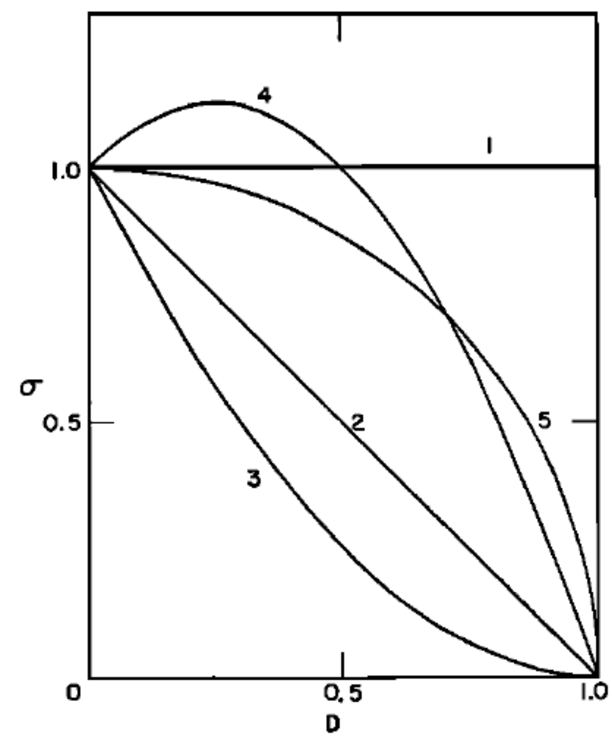
Cohesive Forces over the Fracture

The preceding analysis assumes linear response all over the medium, including the crack-tip vicinity. However, there is no physical material capable to admit infinite stresses or velocities.

Barenblatt (1959) has introduced the concept of cohesive forces in the crack surface. These forces make both the slip rate and the stress concentration ahead the crack tip to be continuous functions (i.e. singularities disappear)

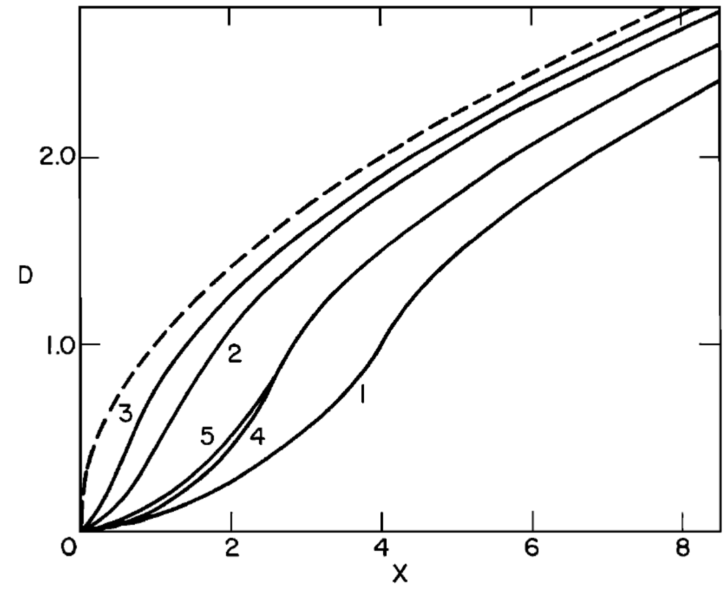
Ida (1972) and then Palmer and Rice (1973) gave a physical sense to Barenblatt's cohesive forces showing that they represent an opposing resistance to dislocation, an energy dissipative process throughout work done over the crack surface against fault tractions.

Ida (1972) introduced the concept of "slip-weakening", so that the fracture resistance in a given point is a function of the displacement discontinuity D. Different laws for the cohesive force σ were proposed, as shown in the figure:



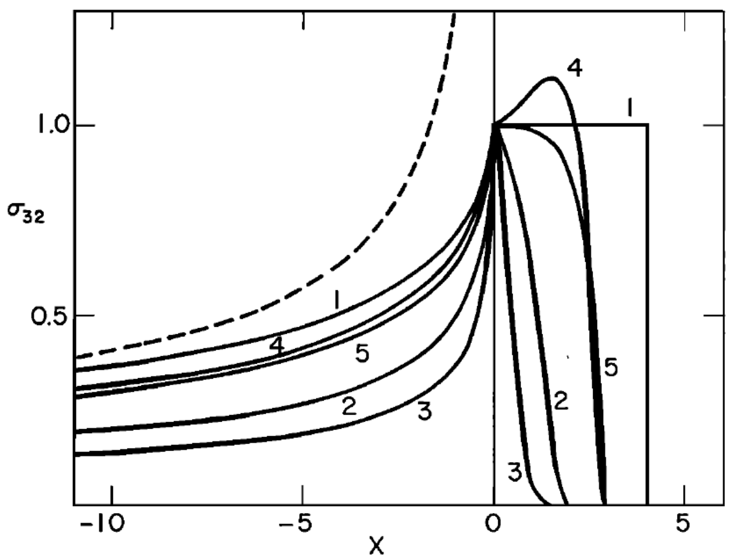
Cohesive Forces over the Fracture

Fracture displacement discontinuities



The associated displacement discontinuities and shear stresses to the above cohesive forces are shown in both figures (Ida, 1972).

Shear tractions inside and outside the fracture



Notice the influence of the cohesive forces on both stresses and the slip functions inside the crack. For smoothly varying cohesive forces in space, the shear traction becomes continuous along the plane where the crack is located.

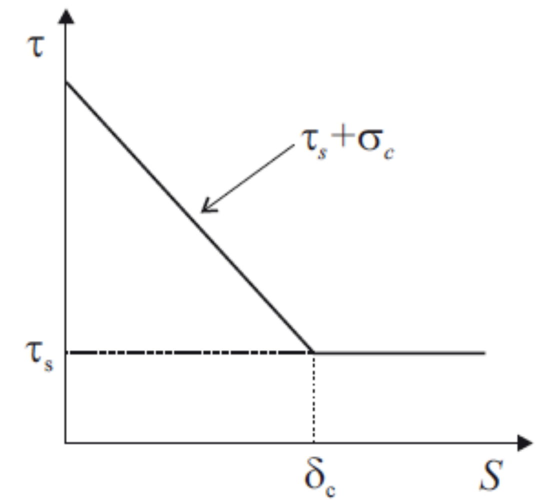
Slip-Weakening Friction

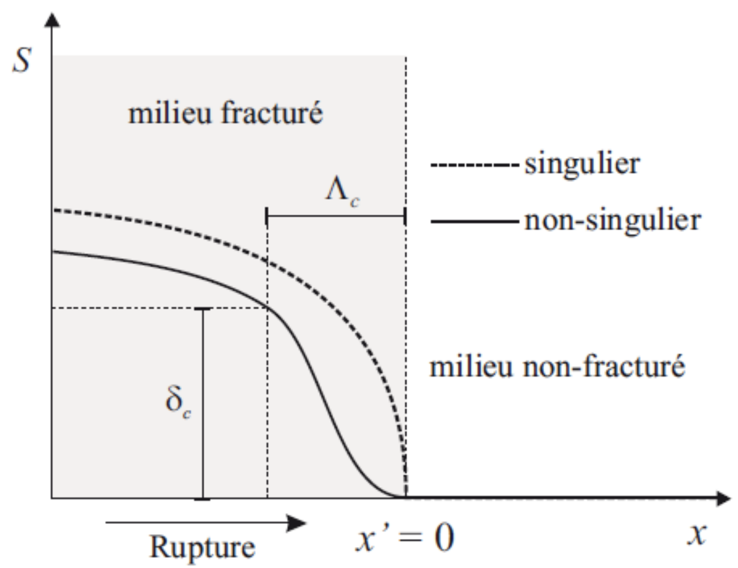
Assuming a linear dependence of the force with slip S (see figure):

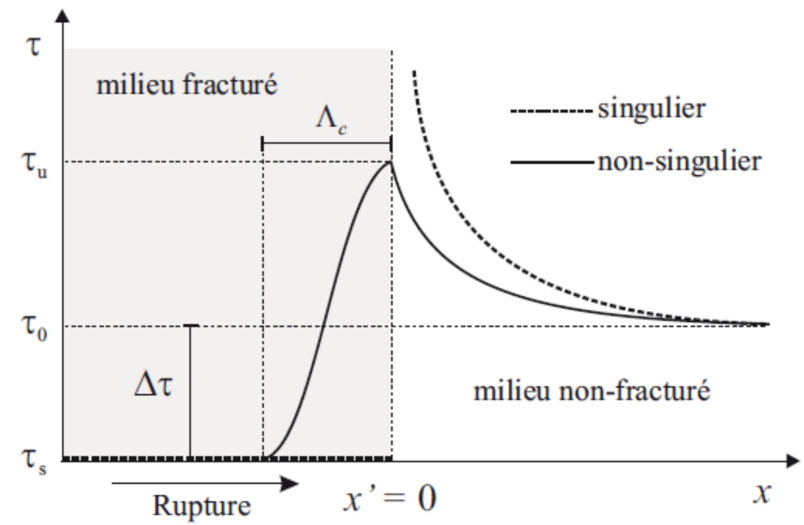
$$\tau(x', t) = \tau_s + \sigma_c[S(x', t)].$$

There exists a cohesive zone behind the crack tip, Λ_c, where friction drops to its dynamic level

Cohesive force as a function of slip







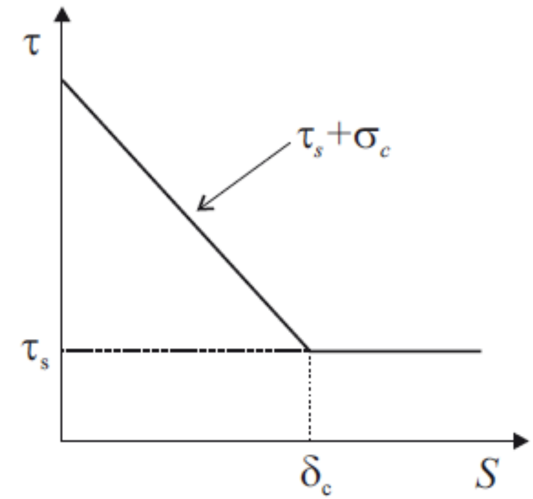
Fracture Cohesive Zone

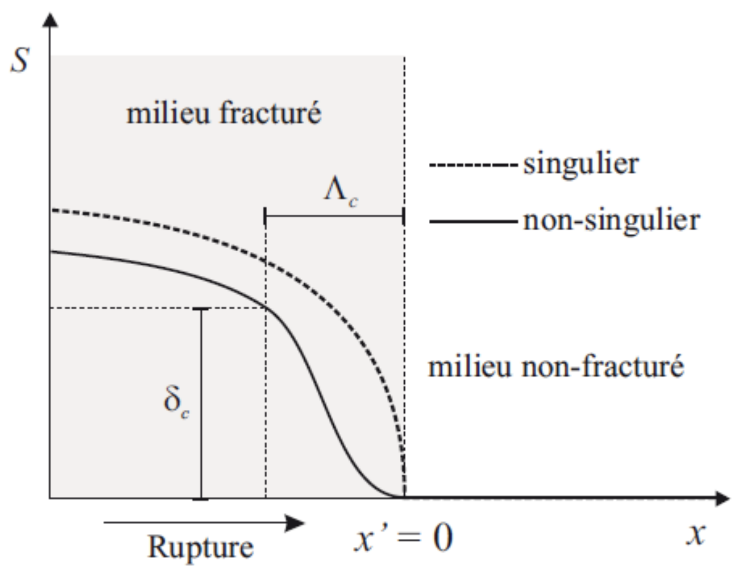
Assuming a linear dependence of the force with slip S (see figure), Ida (1972) showed that the length of the cohesive zone may be approximated as:

$$\Lambda_c = \frac{-k\delta_c}{S'(x_c)},$$

where $S' = \partial S/\partial x.$

Cohesive force as a function of slip





Andrews (2004) showed that such a zone suffers a contraction with rupture propagation distance L so that

$$\Lambda_c = \frac{k\delta_c^2}{L}\left(\frac{\mu}{C\Delta\tau}\right)^2.$$

Day et al. (2005), from energy balance consideration, showed that such Lorentz contraction depends on the fracture mode and rupture velocity.

Fracture Energy Balance

The total energy change in the medium due to a dislocation is given by the Volterra relationship:

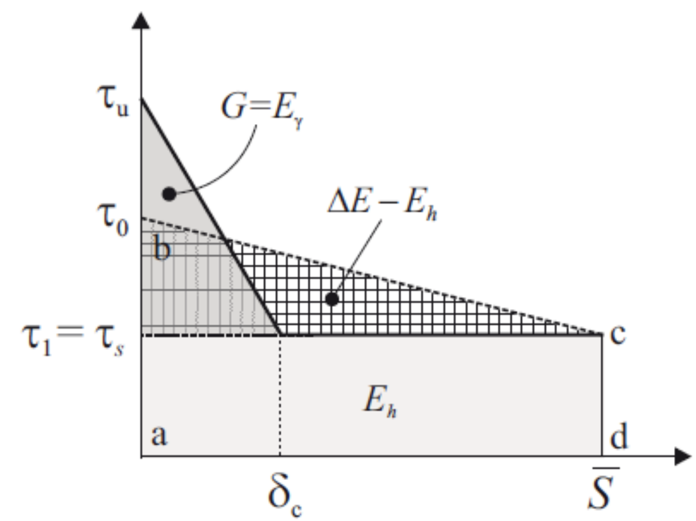
$$\Delta E = -\frac{1}{2}\int_{\Sigma} S_i\left(\tau_{ij}^0 + \tau_{ij}^1\right) n_j d\Sigma.$$

From the principle of energy conservation, the energy partition at the rupture front should satisfy

$$\Delta E - E_h = \underbrace{E_k + E_u}_{E_m} + E_\gamma.$$

where E_h is the heat, E_k the kinetic energy, E_u the elastic energy, E_γ (or G) the fracture energy and E_m the mechanical energy (Husseini, 1977).

Both members of this equation represent the available energy to move the crack



Each term is given as follows:

$$E_k = \frac{1}{2}\iiint_V \rho \left\| v_i \right\|^2 dV$$

$$E_u = \frac{1}{2}\iiint_V \tau_{ij} u_{i,j} dV,$$

$$G = \frac{1}{2}(\tau_u - \tau_s)\delta_c$$