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RESPONSE SPECTRA ON STRATIFIED SOIL

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Abstract

Paper concerns the probability distribution of spectral responses of viscously damped single-degree systems resting on stratified viscoelastic soil. The soil is assumed to rest on a viscoelastic homogeneous half space of rock. Motion arriving at the rock-soil interface is idealized as a stationary Gaussian process. The transfer function for the soil formation is treated independently for each vibration frequency of interest, in order to allow for dependence of viscoelastic parameters on the wave frequency; this is accomplished through use of a matrix formulation. Certain additional approximate results are included.

Nomenclature

a, b, c = constants in power spectral density

a_n, b_n = respectively, amplitude of cosine and of sine terms in the displacements of the nth stratum

B = magnification factor

E = expectation

F = Fourier transform

G = power spectral density

H = thickness of stratum

h = $\zeta\omega$

i = $\sqrt{-1}$

K = function of ζ only

k_n = $\frac{\rho_n v_n}{\rho_{n+1} v_{n+1}} \left(\frac{1 + i\alpha_n}{1 + i\alpha_{n+1}}\right)^{1/2}$

N = number of soil strata plus one

Q = $\max|q(t)|$

q = structural response

R = max r

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$r = \left[(\omega y)^2 + (\dot{y} + hy)^2\right]^{1/2}$

T_n = transmission matrix corresponding to the *n*th soil stratum

V = spectral pseudovelocity

v = velocity of P or S waves

x = displacement

$y = x - x_0$ or $x - x_1$

z = depth measured from rock surface

z_n = depth measured from top of *n*th stratum

α = viscoelastic parameter of soil (a function of ω)

β = reduction factor to take account of damping

δ = Dirac's delta function

ε = unit strain or angular change

ζ = percentage of critical damping

$\eta_n = \nu_n z_n$

μ = elastic (shear) modulus of ground

$\nu = \omega(1 + i\alpha)^{-1/2} v^{-1}$

ξ = parameter defining steady-state ground motion

ρ = mass per unit volume

σ = stress

τ = time

ϕ = autocorrelation function

ψ = basic solution of a dynamic system

ω = circular frequency

ω_0 = undamped natural circular frequency

Simple structure: linear system with one degree of freedom

Introduction

The steady-state one-dimensional vibrations of stratified soil, idealized as viscoelastic, and the deterministic response of undamped simple structures resting on the soil have received attention (e.g. Refs. 1-4). This is true also of structural responses to a wave arriving at the base of the soil formation as a single pulse(5). On the other hand, studies have been made of the distribution of spectral responses to ground motion applied directly to the base of a simple structure and idealized as white noise(6) or as a stationary Gaussian process(7-10). The present paper strives to establish an approximate solution for the probability distribution of spectral responses on soft ground when the disturbance arriving at the rock surface from below is idealized as a stationary Gaussian process. The work is kept within the framework of one-dimensional wave transmission and linear behavior of both the soil and the simple structure.

In the present study advantage will be taken of the fact that a stationary Gaussian process filtered through a linear system gives rise to a motion which is itself a stationary Gaussian process. This type of random motion has received considerable attention; hence, the assumption will be adopted that the motion arriving at the rock surface from below belongs to this kind of process. Accordingly, a brief discussion is presented of the probability distribution of spectral responses to Gaussian processes, beginning with the case of a white-noise disturbance.

Distribution of Responses to Gaussian Processes

Consider a viscously damped linear system with a single degree of freedom whose base is subjected to the accelerogram $\ddot{x}_0(t)$. If the system starts from rest we may write

$$q(t) = \ddot{x}_0 * \psi_q \qquad (1)$$

where q is a structural response (such as absolute acceleration or displacement relative to the ground, ψ_q is the system's basic solution, weighting function, or transfer function for response q (that is, $\psi_q(t) = q(t)$ when $\ddot{x}_0 = \delta(t)$ and δ is Dirac's delta function), and $\alpha * \beta$ signifies $\int_{-\infty}^{t} \alpha(\tau)\beta(t - \tau)\, dt$. Since $\psi_q(t) = 0$ for $t \leq 0$, the lower limit of integration may be replaced with zero. Response spectra are plots of $Q = \max|q(t)|$ as functions of natural period or frequency and it is our aim to calculate the probability distributions of various types of spectral ordinates Q. In particular, $\psi_{\omega_0' y} = \exp(-ht) \sin \omega_0' t$, where $y = x - x_0$, x = absolute displacement of the system, $h = \zeta\omega_0$, ζ = percentage of critical damping, and ω_0 and ω_0' = respectively, undamped and damped natural circular frequencies.

The "response"

$$r = \left[(\omega_0' y)^2 + (\dot{y} + hy)^2\right]^{1/2} \qquad (2)$$

is of special interest. The time derivative at $r^2/2$ is the energy fed to the system per unit time and unit mass, relative to the energy it would be fed were its mass rigidly fixed to the ground.

First let $\ddot{x}_0$ be white Gaussian noise of uniform intensity per unit time in the interval $0 < t \leq s$ where s is the earthquake duration, and $\ddot{x}_0 = 0$ outside this interval. In other words, $\ddot{x}_0$ is a stationary Gaussian process defined by $\phi(t, t-\tau) = a^2 \delta(t)$ if $0 < t - \tau \leq t \leq s$ and $\phi(t, t-\tau) = 0$ if $t \leq \tau$ or $t > s$, a is a constant, and ϕ is the autocorrelation function $E[\ddot{x}_0(t)\ \ddot{x}_0(t - \tau)]$. Let $R = R(h) = \max r(h, t)$. Subject to the condition $2\pi/\omega_0 << s$ the following points have been proved(6).

1. $E[R(0)]$ is proportional to $as^{1/2}$.

2. $E[R(h)] = \beta(hs)\ E[R(0)]$; β can be computed from an infinite series or obtained from available graphs(6) or, with a maximum error of about 4 percent, from the expression

$$\beta \cong \left[\frac{1 - e^{-2hs}}{2hs}\left[0.424 + \ln(2hs + 1.78)\right]\right]^{1/2}$$

(6) or from $\beta \cong (1 + 0.6hs)^{-0.45}$(11). The relation $\beta = \left[1 - \exp(-2hs)\right]^{1/2} (2hs)^{-1/2}$ has also been proposed(10,12) but it does not furnish the ratio of expectations of R(h) to R(0) but the limit of the ratio of these responses when they are both associated with the same probability of being exceeded and that probability tends to zero.

3. The distribution of $R/E[R]$ depends solely on hs.

The quantities ωR, R, and R/ω may be called, respectively, pseudoacceleration, pseudovelocity, and pseudodisplacement. It is easily seen that these quantities are never smaller than $\max|\ddot{x}|$, $\max|\dot{y}|$, and $\max|y|$.

Now consider an earthquake accelerogram $\ddot{x}_0(t)$ of duration s and let $G^2(\omega)$ denote the accelerogram's power spectral density:

$$G^2(\omega) = \frac{2\pi}{s}\left|F(\omega)\right|^2$$

Here $F(\omega)$ is the Fourier transform of $\ddot{x}_0$:

$$F(\omega) = \int_{-\infty}^{\infty} \ddot{x}_0(t)\ e^{-i\omega t}\ dt = \int_0^s \ddot{x}_0(t)\ e^{-i\omega t}\ dt$$

It was assumed in Ref. 6 that, for any response, $E[Q(\zeta, \omega_0)]/E[Q(0, \omega_0)]$ could be taken equal to $\beta(hs)$ as obtained for a white-noise disturbance of some equivalent duration and that the probability distribution of $Q/E[Q]$ would be the same for actual earthquakes as for white motions. It has been shown that these approximations are valid provided the ground motion is nearly Gaussian, its duration considerably exceeds the structure's natural period, this period is much larger than the correlation time of the ground motion, and $G^2(\omega)$ is sufficiently smooth(9). (Near $2\pi/\omega_0 = 0$, for example, the approximation breaks down if q stands for $\ddot{x}$; indeed, $\beta(\infty) = 0$ so that, if $\zeta \neq 0$, this approximation predicts $\ddot{x} = 0$ and hence $\ddot{x}_0 = 0$ at $\omega_0 = \infty$.)

A method has also been developed to pay due consideration to the shape of the power spectrum when the latter is not sufficiently smooth and it is desired to calculate $E[R(\zeta, \omega_0)]$; the assumption that the distribution of $R/E[R]$

is the same as for white noise still holds provided the other conditions quoted above are met, so that it suffices to calculate $E[R]$. The approach can be generalized to responses other than R by combining the reasonings in Refs. 7 and 9, and will be presented here in this more general version.

Clearly, for any ground motion of duration s, $E[\ddot{x}_0(t)] = 0$, since $\dot{x}_0(0) = \dot{x}_0(s) = 0$. A Gaussian process of infinite duration is conceivable, such that its power spectral density be the same as for the earthquake in question. For this new motion let $\phi(t_1, t_2) = E[\ddot{x}_0(t_1)\ \ddot{x}_0(t_2)]$ denote the accelerogram's autocorrelation function. Now consider the case when the Gaussian motion is stationary, that is, $\phi(t, t-\tau) = \phi(\tau)$ depends only on τ for all t. For the new motion,

$$G^2(\omega) = \int_{-\infty}^{\infty} \phi(\tau)\ e^{-i\omega\tau}\ d\tau \qquad (3)$$

(see Refs. 7 and 13-15 for example). Also,

$$E[q^2(\omega_0)] = \int_{-\infty}^{\infty} |F_q(\omega)|^2\ G^2(\omega)\ d\omega \qquad (4)$$

where F_q is the Fourier transform of the basic solution for response q:

$$F_q(\omega) = \int_{-\infty}^{\infty} \psi_q(t)\ e^{-i\omega t}\ dt \qquad (5)$$

Evaluation of $E[Q(\omega_0)]$, when $\ddot{x}_0$ stands for a stationary motion of infinite duration, is meaningless, as for such motions this expectation is infinite. In the case of an actual earthquake, if $\zeta \neq 0$ and if s greatly exceeds both the structure's natural period and the ground motion's correlation time, then for most values of $t > 0$ we shall find the distribution of q almost identical with that of the response to a stationary Gaussian process of infinite duration. If we regard $E[Q]$ as a function of ω_0 it will be nearly proportional to $(E[q^2])^{1/2}$. Hence Tajimi's method, which evaluates $E[Q]$ from

$$\frac{E[Q(\omega_0)]}{E[Q(\infty)]} = \left[\frac{E[q^2(\omega_0)]}{\int_0^{\infty} G^2(\omega)\ d\omega}\right]^{1/2}$$

If $Q(\omega_0)$ stands for the system's maximum absolute acceleration, velocity, or displacement, $Q(\infty)$ represents the maximum ground acceleration, and so on. The method probably gives good results when ζ is not excessively small but cannot be used when $\zeta = 0$, since in that case $E[q^2(\omega_0)] = \infty$.

The following argument leads to a method for calculating $E[Q(\omega_0)]$ that does not break down when ζ tends to zero. Under the conditions described above, the probability distribution of q, without absorbing barriers, tends to become Gaussian with zero mean and therefore to be defined by a single parameter, which is a function of the ground motion. Consequently the distribution of q normalized in terms of this parameter will be independent of the ground motion, the same will be true of the distribution of Q, and since $(E[q^2])^{1/2}$ as well as $E[Q]$ will be proportional to that parameter we may use the responses to a particular set of random motions -- to white noise in particular -- as basis of comparison and write

$$E[Q(\omega_0)] = \left[\frac{\int_{-\infty}^{\infty}|F_q(\omega)|^2\, G^2(\omega)\, d\omega}{\int_{-\infty}^{\infty}|F_q(\omega)|^2\, d\omega}\right]^{1/2} \frac{E[\overline{Q}_a(\omega_0)]}{a} \tag{6}$$

where $E[\overline{Q}_a]$ is the expected value of Q as response to a white disturbance for which $G(\omega) = a$.

For design purposes the response $q = \ddot{x}$ is of special interest. It is easily shown(7,10) that

$$F_{\ddot{x}}(\omega) = \frac{1 - 2i\zeta\omega/\omega_0}{1 - \omega^2/\omega_0^2 + 2i\zeta\omega/\omega_0}$$

$$|F_{\ddot{x}}(\omega)|^2 = \frac{1 + 4\zeta^2\omega^2/\omega_0^2}{(1 - \omega^2/\omega_0^2)^2 + 4\zeta^2\omega^2/\omega_0^2} \tag{7}$$

Also(6), save for very large ω_0 ($\omega_0 s >> 1$), $E[V]$ is practically equal to $E[R]$, where $V\omega_0 = \max|\ddot{x}(t)|$ may be called a spectral pseudovelocity and $E[\overline{R}_a(\omega_0)] = 1.174\, a(\pi s)^{1/2}$. Hence, again save for very large ω_0, $E[V(\omega_0)]$ will be given by Eq. 6 with $F_q = F_{\ddot{x}}$ and $E[\overline{Q}_a(\omega_0)]/a = 1.174(\pi s)^{1/2}$ times a reduction factor, β, that depends on $\omega_0\zeta s$(6). Notice that when $\zeta \rightarrow 0$ the bracketed quantity in Eq. 6 tends to $G(\omega)$.

The above derivations are not limited to simple structures. Eq. 6 for example, is valid when q denotes the response in any generalized coordinate of a linear multidegree system. Further, x_0 may be a column vector representing the set of static displacements induced in the system by the multi-degree-of-freedom base motion, in which case a basic solution Ψ becomes a square matrix which premultiplies $\ddot{x}_0$ in the integrand of Eq. 1, and the Fourier transform of Ψ is also a square matrix.

Distribution of Responses on Soft Ground

Consider a horizontally homogeneous, stratified, viscoelastic soil resting on a semispace of homogeneous viscoelastic rock (Fig. 1). Assume that an ascending horizontal wave* arrives at the rock surface and is such that as t approaches zero the wave accelerations for negative t approach $\delta(t)$; this wave will produce an acceleration $\Psi_1(t)$ at the ground surface. Now let the arriving waves give a random motion at the interface, with power spectral density $G_0^2(\omega)$. In the case of interest $G_0 = G/2$. It is known that filtering a random motion through a linear system transform the power spectral density of the incoming motion into

$$G_1^2(\omega) = |F_1(\omega)|^2\, G_0^2(\omega) \tag{8}$$

where $F_1(\omega)$ is the Fourier transform of the transfer function, and that if the incoming process was Gaussian so will the outgoing motion. Consequently the ground motion in the case which interest us will be Gaussian and may be

* We shall not distinguish between P and S waves, as they are transmitted according to equations that are formally identical.

obtained by setting $F_1(\omega)$ equal to the Fourier transform of the ψ function for $\ddot{x}_1$ in Eq. 8. Combining this result with Eq. 6 we may write

$$E\left[Q_1(\omega_0)\right] = \left[\frac{\int_{-\infty}^{\infty} |F_q(\omega)|^2 |F_1(\omega)|^2 \; G_c^2(\omega) \; d\omega}{\int_{-\infty}^{\infty} |F_q(\omega)|^2 \; d\omega}\right]^{1/2} \; \frac{E\left[\bar{Q}_a(\omega_0)\right]}{a} \tag{9}$$

The earthquake duration to use in calculating $E\left[\bar{Q}_a\right]$ is that of the incoming motion at the rock surface. The numerator inside the brackets of Eq. 9 may be interpreted as the product of $|F_q|^2$ and the filtered spectral density, G_1^2, or as the product of a new transfer function, $|F_q|^2 \, |F_1|^2 = |F_q F_1|^2$, which combines filtering from the interface to the ground surface and from this to the structural response in question.

The problem of determining the effects of soft soil on response spectra reduces therefore to that of valuating $|F_1(\omega)|^2$. This could be done by computing the transfer function and finding its Fourier transform. But we shall find it more convenient to calculate the Fourier transform directly as a result of a study of steady-state harmonic oscillations, especially since the soil's viscoelastic parameters must ordinarily be regarded as functions of ω.

Equivalence of the responses under a Dirac-delta excitation and in steady state may be shown as follows. Consider two linear systems in series, which are defined by the relations

$$\ddot{x}_1(t) = \ddot{x}_0 * \psi_1 \tag{10}$$

$$q(t) = \ddot{x}_1 * \psi_q \tag{11}$$

where x_0 is the motion imposed at the base of the first system, x_1 the displacement at the base of the second one, and q the response of the second system. First let $\ddot{x}_0 = \delta(t)$. From Eq. 10, $\ddot{x}_1 = \psi_1(t)$ and from 11, $q = \psi_1 * \psi_q$. Secondly consider the disturbance $\ddot{x}_0 = \psi_q(t)$. Eq. 10 yields $\ddot{x}_1 = \psi_q * \psi_1$, but in view of the possibility of setting the lower limit of integration equal to zero in the convolution, this is the same as $\psi_1 * \psi_q$. Thus, the response of system 2 to a unit acceleration pulse applied at the base of system 1 is the same as the accelerogram of 1 as response to the disturbance defined by the transfer function of 2. (This reciprocal relation can be extended to the case when x_0, x_1, and q are column vectors and the ψ's are square matrices.)

The amplitude of the acceleration response of an undamped simple system with natural circular frequency ω to a ground motion is equal to ω times the Fourier transform of the motion's accelerogram, since the transfer function for acceleration is then $\omega \sin \omega t$. If the system rests on soft ground, the amplitude of its acceleration response to an ascending wave that arrives at the rock surface with an accelerogram $\delta(t)$ will be $\omega|F_1|$. The amplitude of its acceleration response to the same wave, were the system to rest on rock and were the soil absent, would be 2ω. (The factor 2 stems from the reflection of the pulse at a free surface.) The ratio of amplitudes is the magnification factor for an undamped simple system's response to a unit pulse:

$$B(\omega) = |F_1|/2 \tag{12}$$

Equation 12 exactly supplies the magnification factors for undamped residual spectral ordinates of the ground motion resulting from the arrival of waves of arbitrary shape, since these spectra coincide with the Fourier spectra. Residual spectra provide a lower bound to the response spectra, since the latter represent maximum numerical values of responses, which are at least equal to the amplitudes of residual vibrations. In general, peaks in both types of spectrum nearly coincide(16); this is due to the fact that exceptionally high responses are almost always associated with contribution of the entire ground motion and, in conservative systems, they are therefore bound to occur near the end of the earthquake or after its cessation.

Multiplying the magnification factor for residual spectra by the undamped response spectral ordinates for the free surface of rock may be expected to give a good approximation to the corresponding ordinates on soft ground. It is certainly acceptable for motions of short duration, compared with the structure's natural period, since the maximum response occurs then after the ground motion has ceased. It also tends to become acceptable if the ground motion approaches a stationary stochastic process. For transient disturbances the approximation will always overestimate spectral ordinates on soft ground in the neighborhood of those values of ω for which the computed magnification factor is a maximum, since the residual spectrum gives full magnification to all successive reflections while the maximum response may occur at a finite time, giving smaller magnification. The same cannot be stated about ordinates that correspond to values of ω outside these bands.

Magnification factors for $\zeta = 0$, as computed from the ratio of $E[Q]$ to $E[Q_0]$ obtained from Eqs. 9 and 6, coincide with those for residual spectra.

Calculation of Magnification Factors

With reference to Fig. 1, the equilibrium equation that governs one-dimensional motion of the ground is

$$\frac{\partial \sigma}{\partial z} = \rho \frac{\partial^2 x}{\partial t^2} \tag{13}$$

where the stress $\sigma(t)$ is some functional of the strain $\mathcal{E}(t) = \partial x/\partial z$, and ρ is the density. If $\mathcal{E}$ is of the form $\mathcal{E} = \bar{\mathcal{E}} \exp(i\omega t)$, with $\bar{\mathcal{E}}$ independent of t, and if the stress-strain relation is linear then $\sigma = \bar{\sigma} \exp(i\omega t)$ where $\bar{\sigma} = (1 + i\alpha)\mu\bar{\mathcal{E}}$. Here μ and α are real functions of ω and independent of t.

Under these conditions, for steady-state harmonic disturbance Eq. 13 reduces to

$$(1 + i\alpha)\frac{d^2\bar{x}}{dz^2} + \frac{\omega^2}{v^2}\bar{x} = 0$$

at each of the homogeneous layers, where $x = \bar{x} \exp(i\omega t)$ and $v^2 = \mu/\rho$ (a real function of ω).

With the notation and sign convention of Fig. 1, motion in the nth layer has for solution the expression

$$\bar{x}_n(z,t) = a_n \cos \eta_n + b_n \sin \eta_n$$

where $\eta_n = \nu_n z_n$, $\nu_n = \omega(1 + i\alpha)^{-1/2} v_n^{-1}$, z_n is measured downward from the top of the layer, and the square root is taken such that the real part of ν_n be positive. The functions of ω, ν_n, and α_n characterize the material in the homogeneous layer in question.

Continuity of displacement and stress at interfaces requires that

$$a_{n+1} = a_n \cos \lambda_n + b_n \sin \lambda_n$$

$$b_{n+1} = k_n(-a_n \sin \lambda_n + b_n \cos \lambda_n)$$

where $\lambda_n = \nu_n H_n$,

$$k_n = \frac{\rho_n v_n}{\rho_{n+1} v_{n+1}} \left(\frac{1 + i\alpha_n}{1 + i\alpha_{n+1}}\right)^{1/2}$$

and H_n = thickness of nth layer. The ground surface must be stress free. Hence, $b_1 = 0$.

The above solution applies to $\ddot{x}$ as well as to x; hence, the waves and responses will be treated as though their displacements were the accelerations mentioned in the previous section. Thus, at the rock surface the incoming wave, which is the real part of $(1/2)(a_N + b_N/i)\exp(i\omega t)$, is stipulated as $\sin \omega t$, that is, as the real part of $-i \exp(i\omega t)$. Therefore,

$$a_N - ib_N = -2i \tag{14}$$

Now define the matrix

$$T_n = \begin{vmatrix} \cos \lambda_n & \sin \lambda_n \\ -k_n \sin \lambda_n & k_n \cos \lambda_n \end{vmatrix}$$

so that

$$\begin{vmatrix} a_N \\ b_N \end{vmatrix} = T_{N-1} T_{N-2} \cdots T_1 \begin{vmatrix} a_1 \\ b_1 \end{vmatrix}$$

With the additional definition

$$\bar{\xi} = \begin{vmatrix} \xi_1 \\ \xi_2 \end{vmatrix} = T_{N-1} T_{N-2} \cdots T_1 \begin{vmatrix} 1 \\ 0 \end{vmatrix}$$

we obtain

$$\begin{vmatrix} a_N \\ b_N \end{vmatrix} = a_1 \bar{\xi}$$

because $b_1 = 0$. Therefore Eq. 14 reduces to $a_1(\xi_1 - i\xi_2) = -2i$, or $a_1 =$

$2/|\xi_2 + i\xi_1|$. The steady-state ground-surface motion is the real part of $a_1 \exp(i\omega t)$, whose amplitude is the Fourier amplitude spectrum, $2/|\xi_1 + i\xi_2|$, and the magnification factor sought, according to Eq. 12, is

$$B(\omega) = \frac{1}{|\xi_1 + i\xi_2|} \tag{15}$$

When the soil's internal damping is neglected, λ_n and k_n become real and Eq. 15 may be interpreted in the following way. Vector $\bar{\xi}$ is found by taking the unit vector along an arbitrary ξ_1 direction, rotating it through an angle λ_1, amplifying (or contracting) it, in proportion to k_1 in the direction ξ_2, perpendicular to ξ_1, rotating the resulting vector through an angle λ_2, and so on, N-1 times (Fig. 2). The magnification factor B equals the reciprocal of the vector's amplitude at the end of the process. This interpretation leads to Takahasi's(2) graphical solution.

In the case of a single undamped homogeneous layer Eq. 15 may be written in the well-known form

$$B(\omega) = (k_1^2 \sin^2 \lambda_1 + \cos^2 \lambda_1)^{-1/2}$$

where $k_1 = (\rho_1 v_1/\rho_2 v_2)$ and $\lambda_1 = \omega H_1/v_1$.

Application

The methods described in this paper are applied to the example in Fig. 3. The power spectral density for the motion on rock, were the soil absent, might be assumed given by the expression

$$G^2(\omega) = \frac{a^2(1 + 4b^2\omega^2/c^2)}{(1 - \omega^2/c^2)^2 + (2b\omega/c)^2} \tag{16}$$

which has been proposed by Tajimi(7) on the basis of work by Kanai(4). The constants in this expression have been taken as $a^2 = 500/s$, $b^2 = 0.205$, and $c^2 = 242\ \text{sec}^{-2}$ which are representative for earthquakes on hard ground in the west coast of the U.S.(10), with $s = 20$ sec. However, according to Eq. 6, when $\zeta = 0$ $E[Q(\omega)]$ is proportional to $G(\omega)$ and, if Eq. 16 is taken to give the square of a quantity proportional to the expected acceleration of spectrum on rock, it fails to satisfy the obvious requirement that $\omega^{-1} E[Q(\omega)]$ remain finite as ω tends to zero. Consequently Eq. 16 will be replaced with

$$G^2(\omega) = \frac{(2ab\omega/c)^2}{(1 - \omega^2/c^2)^2 + (2b\omega/c)^2} \tag{17}$$

which is consistent with the expression for expected acceleration spectra proposed in Ref. 17. (It should not be construed from the above remarks that Eq. 17 more closely represents the power spectral density of actual, earthquake motions on firm ground than does Eq. 16. The opposite is the case. But the approximations involved in Eq. 6 lead to the necessity of using a fictitious spectral density that will give the correct expected acceleration spectra for zero damping.)

With these data, $E[V_0(\omega_0, 0)]$, $\beta E[V_0(\omega_0, 0)]$, and $E[V_0(\omega_0, \zeta)]$, $\zeta = 0.05$, were calculated through numerical integration using Eqs. 6 and 7; here V_0 is a spectral pseudovelocity in the sense that $\omega_0 V_0 = \max|\dot{x}(t)|$. Results are shown in Fig. 4. It is seen that $\beta E[V_0(\omega_0, 0)] \cong E[V_0(\omega_0, \zeta)]$ except for relatively small values of $2\pi/\omega_0$.

Next the theory on multiple wave reflection was used to compute $B(\omega)$. The product $B(\omega_0) E[V_0(\omega_0, 0)] = E[V_1(\omega_0, 0)]$ is shown in Fig. 5. Also shown are the curves for $\beta E[V_1(\omega_0, 0)]$ and $E[V_1(\omega_0, \zeta)]$, the latter obtained from Eq. 9. Comparison of these two curves evaluates the assumption that the reduction factor β (for responses to white noise) applies to the motion at the ground surface. It is seen that the assumption holds for relatively long natural periods but is untenable in the rest of the expected spectrum, especially in intervals of pronounced curvature.

Concluding Remarks

The assumption that earthquakes are stationary Gaussian processes is fruitful in that it permits establishing methods based on the premise that expected response spectra (spectra of maximum numerical value of responses) are proportional to the square root of the expected squared response at any given instant. The idealization precludes direct calculation of expected response spectra, as these are infinite for stationary random processes of infinite duration. The central portion of the present work makes use of this premise and supplements it with known approximate results for the distribution of spectral responses to stationary white noise of finite duration.

Other approximate treatments are also considered, such as the assumption that magnification factors for residual spectra should be applicable to response spectra; this particular assumption is of interest because of its simplicity, as the magnification factors are easily found from the Fourier transform of the soil's transfer function, and they apply rigorously to the residual spectra of arbitrary ground motions. It is concluded that the approximation is not altogether satisfactory throughout the range of interest in spectra. It is found that effects of the structure's damping are particularly pronounced near the locally dominant periods of the ground.

The most drastic assumptions made in this study concern the one-dimensional nature of the ground motion and the linear behavior of the soil. The first assumption is to a large extent justified by Snell's law when the soil is stratified nearly parallel to the ground surface. Under these conditions S- and P-wave motion are, respectively, nearly parallel and normal to the surface. Unevenness of this surface and of interfaces as well as small horizontal heterogeneity of strata can probably be taken into account, approximately, through an increase in the soil's equivalent internal damping by some increasing function of the wave frequency.

The assumption of linear behavior is acceptable up to a certain earthquake intensity. This upper limit is a function of local conditions. It is quite high for most cohesive soils but almost zero for noncohesive ones, except when they are effectively confined by cohesive materials. Owing to this limitation the linear theory of multiple wave-reflection has received little credit in extensive areas of the world while it is of much use for the design of buildings on soft lacustrine clay and other cohesive formations.

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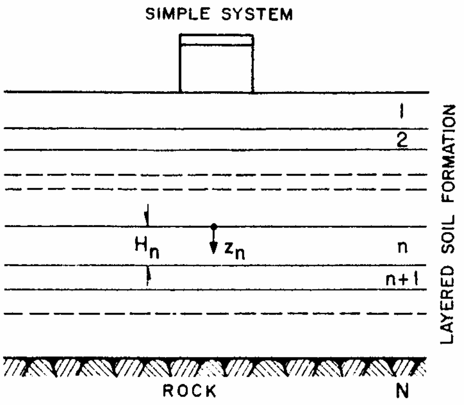

FIG.1 SECTION OF SUBSOIL

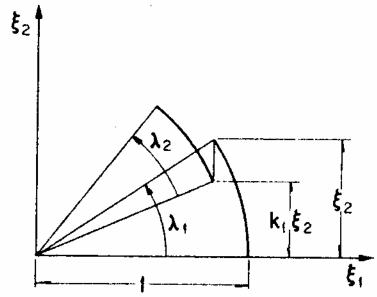

FIG.2 GRAPHICAL SOLUTION

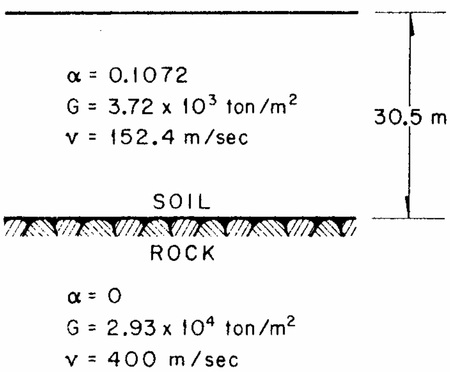

FIG.3 PARAMETERS USED IN THE NUMERICAL EXAMPLE

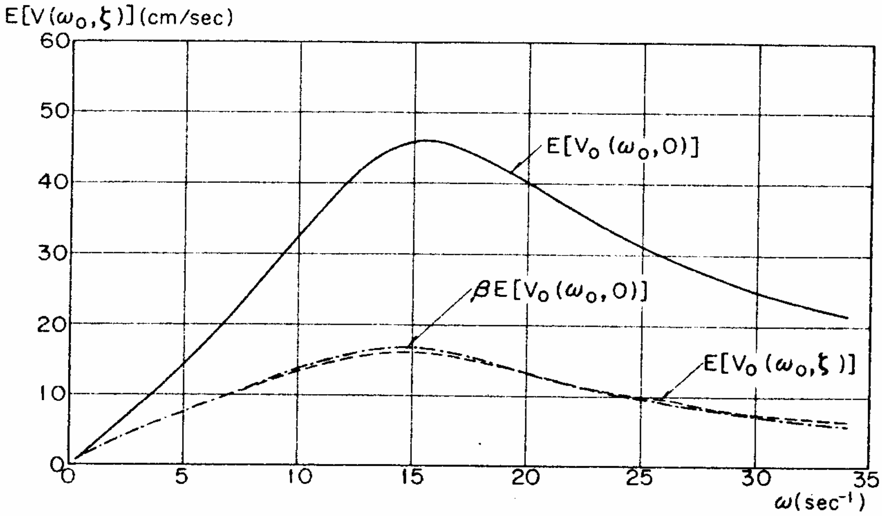

FIG. 4 APPLICATION RESPONSE SPECTRUM ON ROCK

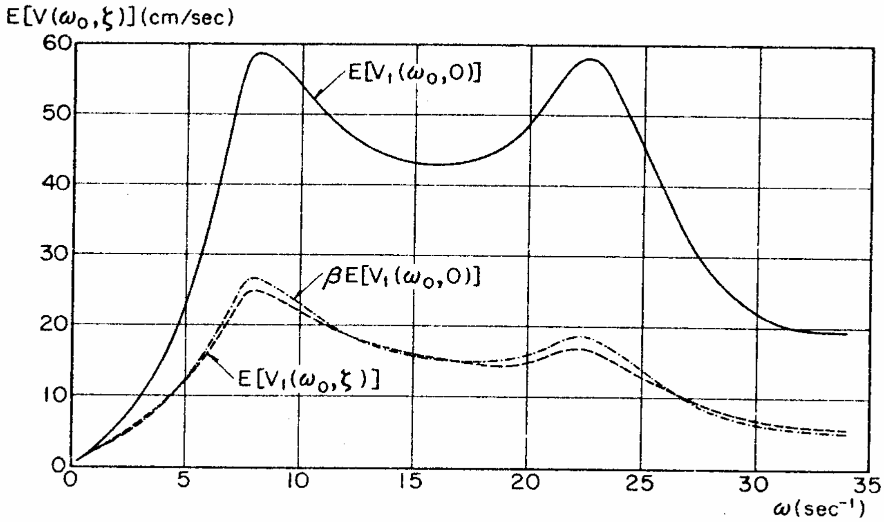

FIG. 5 APPLICATION RESPONSE SPECTRUM ON SOFT GROUND