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HYDRODYNAMIC PRESSURES GENERATED BY VERTICAL EARTHQUAKE COMPONENT

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SYNOPSIS

In a recent study, Chopra [1], using vertical rigid motions of the bottom of a reservoir, found that hydrodynamic pressures due to vertical accelerations of the El Centro earthquake were much greater than usually admitted before. His conclusion is difficult to accept and generalize because, if it were true, several existing dams would have suffered important damage.

This paper covers a more complete analysis of the same problem, including the effects of non-simultaneous arrival of seismic waves to the bottom of the reservoir. Results are compared to those found in previous work.

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ABSTRACT

In this paper the effect on the hydrodynamic pressures of the finite velocity of propagation of the earthquake is considered. The vertical component of the acceleration is the only one taken into account. The fluid compressibility is included in the analysis. As an example the method is applied to the El Centro, Cal. May 18, 1940, earthquake. The results are compared with those of Chopra. The main conclusion of the paper is that hydrodynamic pressures are strongly influenced by the direction of approach and velocity of propagation of the earthquake. Further research is needed to elucidate this matter. Additional results are presented for uniform vertical ground motion taking into account the refraction of pressure waves at the bottom.

INTRODUCTION

When a dam is subjected to an earthquake, dynamic motions and stresses are induced on the soil, the dam, and the contained water. The dynamic stresses on the dam body depend on the interaction or dynamic coupling between the dam, the water, and the soil. This makes very complicated the study of the simultaneous response of the dam, water, and soil system. Therefore, the response of the dam and the water is studied separately, without considering their mutual interaction, and adding to the dynamic response of the isolated dam, the hydrodynamic pressures obtained treating the dam as rigid.

In most studies of hydrodynamic pressures, the effects of the vertical and horizontal components of the earthquake are considered separately. The maximum hydrodynamic pressures on the dam due to horizontal ground motions have acceptable values [1,2]; but the maximum dynamic water pressures due to vertical acceleration of the bottom is unbounded; a previous work, [1] gives high values for small height dams.

In this study it is intended to obtain a better understanding of the hydrodynamic response to vertical ground motions.

ASSUMPTIONS

In order to find the hydrodynamic pressures on the dam, the following assumptions will be used:

Geometrical properties

1. The upstream face of the dam is flat and vertical, (fig 1)
2. The reservoir length extends to infinity in the upstream direction
3. The depth of the reservoir is constant
4. The reservoir has a rectangular transversal section

Behaviour, and dynamic effects

5. The dam is rigid
6. The water is linearly compressible, and the viscosity effects are

neglected

7. The liquid has small displacements and its movement is irrotational
8. Surface waves are neglected
9. The water motion is bidimensional, i.e. it is the same for any vertical plane parallel to the axis of the reservoir, (fig 1)
10. The reservoir does not return energy to the ground, i.e. refraction does not occur at the bottom

The vertical earthquake component

11. In this paper only vertical displacements of the bottom are considered The movement may be not uniform and agrees with the following assumption
12. The earthquake propagates with constant horizontal velocity in the direction of the axis of the reservoir. Two senses of motion are considered. Firstly, when the earthquake moves from the dam face towards the interior of the reservoir, and secondly, when the earthquake moves from the interior of the reservoir towards the dam face (fig 1)

In previous papers [1,3], in which water compressibility is considered, the ground was assumed to be rigid and all its points were supposed to move in a uniform manner. This turns out to be a limit case of the formulation presented in this paper. It is obtained if the horizontal propagation velocity of the earthquake is taken to be infinite. Under such assumption the hydrodynamic pressures only depend on the vertical coordinate and time. According to the way in which the problem is presented here, hydrodynamic pressures depend on the horizontal coordinate as well.

The neglect of surface waves is sufficiently accurated in the analysis of uniform vertical motion of the bottom [1].

The ground will be assumed to be rigid in all cases for which the propagation velocity of the earthquake is finite. Only when this speed is infinite, the effect of the ground compressibility will be taken into account. A detailed probabilistic study for this case can be found in reference 4.

Assuming a finite horizontal velocity of propagation for the earthquake allows to include more realistic conditions than those considered in previous works [1,3], and makes clear the real meaning of the important hydrodynamic pressures obtained when uniform motion of the rigid ground is assumed [1].

STATEMENT OF THE PROBLEM

Under the assumptions stated previously the hydrodynamic pressure is governed by the wave equation.

Let $p(x,y,t)$ denote the hydrodynamic pressure, then

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} - \frac{1}{v_0^2}\frac{\partial^2 p}{\partial t^2} = 0 \qquad (1)$$

where x,y are the coordinates (fig 1), and $v_0 = 1438$ m/sec is the velocity of sound in water, and t is the time.

The boundary conditions at the upstream dam face $x = 0$, and at the free surface $y = H$, (fig 1) are

$$\frac{\partial p}{\partial x}(0,y,t) = 0 \quad , \quad p(x,H,t) = 0 \qquad (2),(3)$$

In the bottom of the reservoir, when no energy losses are considered, the boundary condition is

$$\frac{\partial p}{\partial y}(x,0,t) = -\frac{\gamma_0}{g} A\left(t \mp \frac{x}{v_s}\right) \qquad (4)$$

where $A(t)$ is the vertical ground acceleration of the earthquake, γ_0 = the unit weight of water = 1 ton/m^3, g = the acceleration of gravity = 9.81 m/sec^2, and v_s is the horizontal velocity of the seismic waves. When $t - x/v_s$ is used, the earthquake leaves the dam in the upstream direction, and when $t + x/v_s$ is used, the earthquake comes from the reservoir up to the dam in the downstream direction; in this case the earthquake starts at $x = +\infty$

It is assumed that the water in the reservoir is at rest before the arrival of the earthquake, i.e.

$$p = \frac{\partial p}{\partial t} = 0 \qquad (5)$$

before the earthquake arrives.

Now an additional assumption is included: the earthquake travels with supersonic velocity, that is

$$v_s > v_0 \qquad (6)$$

This relation is accomplished by pressure waves of an earthquake on rock.

METHOD OF SOLUTION

The solution was obtained in two steps. First, the solution for an infinite plane was established, and called "the auxiliary solution". In the second step, using the superposition principle, by means of the method of images, the desired solution, which satisfies the boundary condition of eqs 2 and 3 was constructed. The desired solution was called "the actual solution".

Furthermore, the solution were obtained for an instantaneous unit impulse function corresponding to a concentrated acceleration $\delta(t)$, and then the summation or convolution of the effects of the various impulses constituting the earthquake accelerogram, was accomplished.

The often called "Dirac Delta function", denoted by $\delta(t)$, has the same property as a concentrated effect, thus: $\delta(t)=0$ for $\neq 0$, $\delta(t)\neq 0$ for $t=0$ such that

$$\int_a^b \delta(t)\,dt = 1 \quad , \quad a<0<b \tag{7}$$

Also the unit step function U will be used, which can be defined as: $U(t)=0$ for $t<0$, $U(t)=1$ for $t\geq 0$, and satisfies

$$U(t) = \int_{-\infty}^{t} \delta(t)\,dt \tag{8}$$

The auxiliary solution

a. Case when the earthquake leaves the dam in the upstream direction

This solution is based in the "radiation problem" in two dimensions which consists in a fixed point source of radiation in $x=\xi$ and $y=0$ (fig 2a), and is given by [5,6]

$$\frac{1}{2\pi}\int_0^{t-R/v_0} \frac{f(\theta)\,d\theta}{\sqrt{(t-\theta)^2-(R/v_0)^2}} \qquad \text{for } R<v_0 t \tag{9}$$

and zero when $R\geq v_0 t$, where θ integration parameter, and $R=\sqrt{(\xi-x)^2+y^2}$, and $f(t)$ is the function of intensity of radiation [6].

Now, when $f(t)$ travels with a velocity v_s leaving the origin (see ec 4), using (fig 2b), $f(t)$ can be expressed [7] as:

$$f(t) = 2\,\frac{\gamma_0}{g}\int_0^{v_s t} A(t-\xi/v_s)\,d\xi \tag{10}$$

and with eqs 9 and 10 the auxiliary solution at $x=0$ is

$$\bar{p}'(y,t) = \frac{1}{\pi}\int_0^{v_s t}\int_0^{t-r/v_0} \frac{\gamma_0}{g}\,\frac{A(\theta-\xi/v_s)\,d\theta\,d\xi}{\sqrt{(t-\theta)^2-(r/v_0)^2}} \tag{11}$$

it gives the hydrodynamic pressures at the vertical axis $x=0$, in this case. Here, $r=\sqrt{\xi^2+y^2}$

The solution due to an instantaneous impulse was obtained using the property given by eq 7, in the form

$$A(t) = \int_0^t A(t-\tau)\,\delta(\tau)\,d\tau$$

and therefore

$$\bar{p}'(y,\ ) = \int_0^t \frac{\gamma_0}{g}\,A(t-\tau)\,\bar{p}'_\delta(y,\tau)\,d\tau \tag{13}$$

where

$$\bar{p}'(y,\tau) = \frac{1}{\pi}\int_0^{\tau v_s}\int_0^{\tau-r/v_0} \frac{\delta(\theta-\xi/v)\,d\theta\,d\xi}{\sqrt{(\tau-\theta)^2-(r/v_0)^2}} = \frac{1}{\pi}\int_0^{\tau v_s} \frac{d\xi}{\sqrt{(\tau-\xi/v_s)^2-(r/v_0)^2}} \tag{14}$$

Here $r=\sqrt{\xi^2+y^2}$ and eq 7 were used. When $v_s>v_0$ eq 14 gives

$$\bar{p}'_{\delta}(y,\tau) = \frac{v_s}{\pi\beta} \arctan \beta \sqrt{1 - \left(\frac{y}{v_0 \tau}\right)^2}$$

with

$$\beta = \sqrt{(v_s / v_0)^2 - 1}$$

b. Case when the earthquake is coming to the dam.

In this case the earthquake starts in $x=+\infty$ and reaches the dam face in $t=0$ with stationary or standing pressure waves behind it, if $v_s > v_0$. This pressure waves are given by

$$\hat{p}'_{\delta}(y,\tau) = \frac{v_s}{\beta} U\left(\tau - \frac{y\beta}{v_s}\right)$$

where U is defined by eq 8. In order to construct the actual solution, the pressure waves reflected at the dam face, must also be considered. They are given by the transitory solution which is obtained in the case when the earthquake leaves the dam.

<u>The actual solution</u>

a. Case when the earthquake leaves the dam

Using the method of images, in order to satisfy the rigid dam condition (fig 3), the values given by eq 15 must be duplicated because the effects of the source and of the image are additive.

In order to satisfy eqs 2 and 3 antisymmetrical images with respect to $y = H$ must be considered (fig 4); it gives the actual solution in the form:

$$\bar{p}_{\delta}(y,\tau) = \frac{2 v_s}{\pi\beta} \sum_{n=0}^{N} (-)^n \left\{ \arctan \beta \sqrt{1 - \left(\frac{2nH+y}{v_0 \tau}\right)^2} - \arctan \beta \sqrt{1 - \left(\frac{2(n+1)H - y}{v_0 \tau}\right)^2} \right\} \quad (18)$$

where $0 \leq y \leq H$ and $N =$ maximum integer of $[(v_0 t + y)/2H]$. Now eq 13 changes to

$$\bar{p}(y,t) = \int_0^t \frac{\gamma_0}{g} A(t-\tau)\, \bar{p}_{\delta}(y,\tau)\, d\tau$$

where $\bar{p}(y,t)$ represents the desired pressures when the earthquake leaves the dam

b. Case when the earthquake is coming to the dam.

As in the previous case, the rigid dam condition duplicates the values of eq 17. Substracting from the value so obtained the pressure waves corresponding to case a, the actual solution takes the form

$$\hat{p}_{\delta}(y,\tau) = \frac{2 v_s}{\beta} \sum_{n=0}^{N} (-)^n \left\{ U\left(\tau - \beta \frac{2nH+y}{v_s}\right) - U\left(\tau - \beta \frac{2(n+1)H - y}{v_s}\right) \right\} - \bar{p}_{\delta}(y,\tau)$$

where N is the same as in eq 19, and $\bar{p}(y,\tau)$ is given by eq 18.

In eqs 18 and 20, refractions of the pressure waves are not considered. One way to include them, at least approximately because of their dependence of the angle of incidence at the bottom, is to substitute the coefficient $(-)^n$ in eqs 18 and 20 by

$$\left(\frac{1+C_r}{2}\right)(-C_r)^n \qquad (21)$$

where $C_r=(k-1)/(k+1)$ is a reflection coefficient that considers the energy losses of each reflection of the pressure waves when they reach the bottom, $k=(v_s\gamma_s)/(v_0\gamma_0)$, where γ_s unit weight of rock.

c. Case of uniform vertical motion of the bottom.

Using eq 21 in eqs 18 and 20 and allowing C_r be constant, when $v_s \rightarrow \infty$; both equations coincide with a previous solution which includes the refracted pressure waves [4], and can be written as:

$$\overset{\infty}{\bar{p}}_\delta(y,\tau)=v_0\left(\frac{1+C_r}{2}\right)\sum_{n=0}^{N}(-C_r)^n\left\{U\left(\tau-\frac{2nH+y}{v_0}\right)-U\left(\tau-\frac{2(n+1)H-y}{v_0}\right)\right\} \qquad (22)$$

When $C_r=1$, eq 22 is equivalent to the equations used in ref 1.

Equation 19 should be used to compute the hydrodynamic pressures, where $\bar{p}_\delta$ corresponds to the desired response (eqs 18, 20 or 22).

<u>Numerical methods used in digital computer</u>

To compute the transient hydrodynamic pressures, the total lateral force and the total overturning moment in the dam face, a FORTRAN program was developed.

The computation was made in two steps. First, the transfer function or the response function due to a unit concentrated impulse $\delta(t)$ is computed and tabulated. This function will be denoted by $G_\delta(t)$. Second, the convolution

$$R(t) = c\int_0^t A(t-\tau)G_\delta(\tau)\,d\tau$$

was computed, where $R(t)$ is the desired response, and c is the coefficient depending on the function used $G_\delta(t)$.

The total force was obtained using

$$G_\delta(\tau) = F_\delta(\tau) = \int_0^H p_\delta(y,\tau)\,dy \qquad (24)$$

and the total overturning moment was computed by means of

$$G_\delta(\tau) = M_\delta(\tau) = \int_0^H y\, p_\delta(y,\tau)\,dy \qquad (25)$$

Eqs 24 and 25 were computed using the analytical results of term-by-term integration and, then, the summation of the corresponding p_δ, (eqs 18, 20 or 22) was done. Equation 23 was computed using a linear approximation between two succesive data points of the accelerogram, (fig 5), and the effect of this pulse was computed by means of

$$\Delta R = \frac{\Delta\tau}{G}\left\{(a+2b)A+(2b+c)B\right\} \tag{26}$$

where A and B are the accelerations at the end of the interval, and a, b and c are shown in fig 5 b; they were computed using four points of Lagrange interpolation in the function $G_\delta(\tau)$ tabulated at equal intervals. Eq 26 corresponds to a cubic parabola approximation. Eq 23 was computed using the summation

$$R(t) = c\sum\Delta R$$

which was done for each time t, from $\tau = 0$, to $\tau = t$

HYDRODYNAMIC RESPONSE

In what follow the basic features of the hydrodynamic response are described for a unit vertical impulse $\delta(t)$ and for the vertical component of the El Centro earthquake.

In fig 6 the transfer functions F_δ and M_δ occurring in eqs 24 and 25, are shown for the case when the impulse leaves the dam, (fig 1). They were computed by using $v_s/v_0 = 3$, H = 100 metters, in eq 18. In dotted lines (fig 6) is also included the case when $v_s/v_0 = \infty$, which corresponds to those used in a previous work [1], (eq 22 with $C_r = 1$). Note that the functions of fig 6 when $v_s/v_0 = \infty$ and when $v_s/v_0 = 3$ are not in phase. In solid line an apparent damping is present. If v_s/v_0 is increased, the transfer function approaches the case $v_s/v_0 = \infty$. If v_s/v_0 is decreased the decay with time increases and the transfer function has smaller values.

When the impulse is coming downstream to the dam, see eq 20, the transfer function does not decay with time because the step function U has a constant value, and $\bar{p}_\delta$ of eq 20 decreases with time in view of fig 6. If v_s/v_0 is increased without limit, the transfer function approaches from above to the case $v_s/v_0 = \infty$. But now, if v_s/v_0 is decreased, in view of the factor v_s/β of eq 20, the transfer function increases when $v_s/v_0 > 1$, and therefore no upper bound exists for the values of the transfer function, when v_s/v_0 is nearly unity.

The hydrodynamic responses were computed using the vertical accelerations of the El Centro earthquake. In fig 7 it is shown the accelerogram of the first five seconds of the earthquake. Its duration was 31.71 sec. with a maximum vertical acceleration of 0.28 of the gravity, reached at 0.97 sec.

In table 1 are given the maximum dynamic/static values for both total force and total overturning moment on the dam face for $v_s/v_0 = 3$ and for $v_s/v_0 = \infty$ The time of maximum response is given in parenthesis. The convolution was made for the first 8 sec of the earthquake. The values of table 1 are shown in fig 8. The lower maximum pressures occur when the earthquake leaves the dam in the upstream direction, but when the earthquake comes downstream to the dam, the maximum pressures rice to high values. The case $v_s/v_0 = \infty$ is between the extreme cases and corresponds to a previous study [1]. In figs 9 to 12 the responses with time are shown for the first five seconds when the earthquake leaves the dam in the upstream direction.

In fig 13 are shown the maximum hydrodynamic pressures for uniform vertical ground motion with various values of the reflection coefficient C_r in ec 22. The results were obtained during the first eight seconds of the earthquake. In the same figure, the pressures decrease with C_r . A typical value of C_r for water and rock [4], ($\gamma_0 = 1$ ton/m^3 , $v_0 = 1438$ m/sec $\gamma_s = 2.8$ ton/m^3 , $v_s = 5000$ m/sec) is $C_r = 0.815$; (ec 21). In fig 13 only $v_s/v_0 = \infty$ was used, this result must be between the values of $v_s/v_0 = 3$, (fig 8), and also a proportional reduction of maximum pressures can be expected if the reflections are included in the cases with $v_s/v_0 = 3$

CONCLUSIONS

In this paper the following facts have been established.

When the horizontal velocity of the earthquake is three times the velocity of sound in water, the maximum pressures have relatively small values if the earthquake travels in the upstream direction, but if the earthquake travels in the downstream direction the maximum pressures are very big.

- When the velocity of propagation of the earthquake is infinite, the maximum hydrodynamic pressures are between the former cases.

- When energy losses are included, due to refraction of the pressure waves at the bottom of the reservoir, the maximum pressures are greatly reduced.

From these facts the following conclusions follow:

- <u>Hydrodynamic pressures are strongly influenced by the direction of approach and velocity of propagation of the earthquake</u>.

- <u>Further research is necessary to elucidate this matter</u>

ACKNOWLEDGEMENT

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REFERENCES

1. A.K. Chopra, "Hydrodynamic pressures on dams during earthquakes," Journal of the Engineering Mechanics Division, ASCE, Vol. 93, No. EM 6, Proc. Paper 5695 (Dec. 1967), 205-223.

2. J.I. Bustamante and A. Flores "Water pressure on dams subjected to earthquakes, "Journal of the Engineering Mechanics Division,"ASCE, Vol. 92, No. EM 5, Proc. Paper 4954 (Oct. 1966), 115-127.

3. S. Kotsubo, "External forces on arch dams during earthquakes (A study on the aseismic design of arch dams - Part I)", Memoirs of the Faculty of Engineering, Kyushu University, Fukuoka, Japan, Vol. 20, No. 4 (1961), 327-366.

4. E. Rosenblueth, "Presión hidrodinámica en presas debida a la aceleración vertical con refracción en el fondo," II Congreso Nacional de Ingeniería Sísmica, Veracruz, Ver., Mexico, (May, 1968).

5. H. Lamb, "Hydrodynamics", Sixth Ed. Dover Publications (1945), Arts. 194 to 197, pp. 293-299.

6. R. Courant and D. Hilbert, "Methods of mathematical physics," Vol. II, Interscience Publishers, Inc., New York (1962), p. 205 to 207, p. 695 and following.

7. A. Flores Victoria, I. Herrera and C. Lozano, "Presión Hidrodinámica generada por la componente vertical de un sismo," II Congreso Nacional de Ingeniería Sísmica, Veracruz, Ver., Mexico (May, 1968).

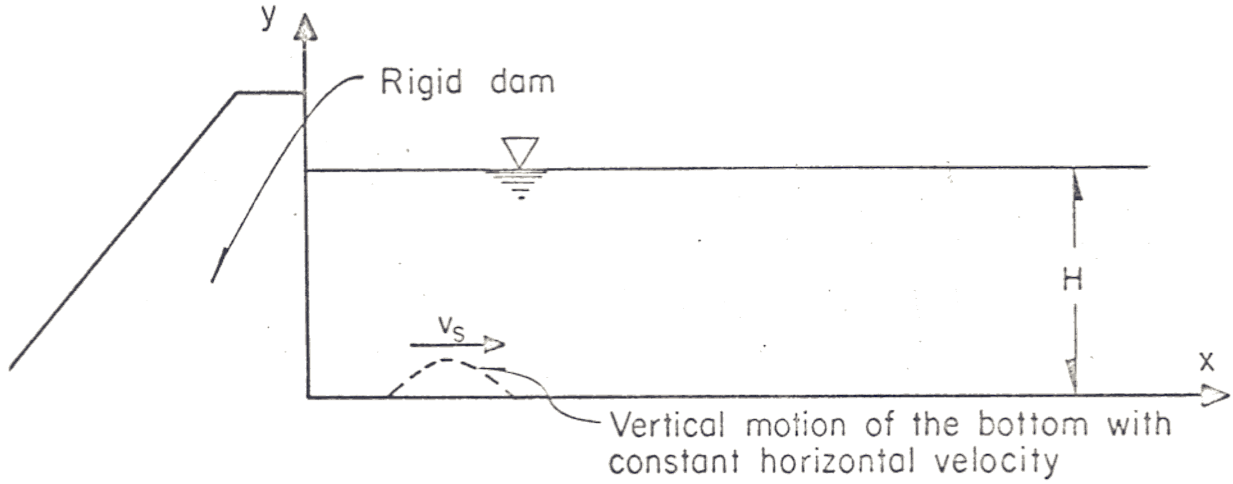


Fig 1 Idealized system

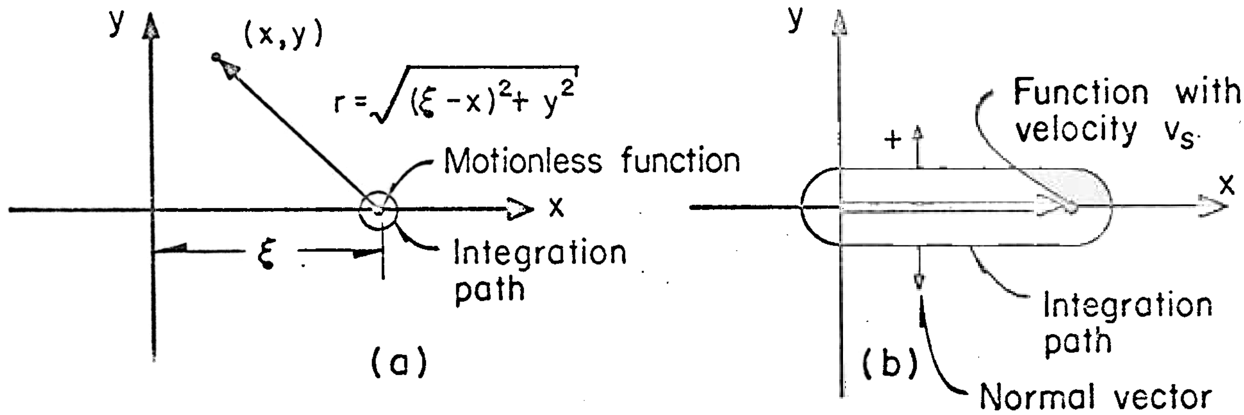


Fig 2 Localization of intensity of radiation functions in infinite plane

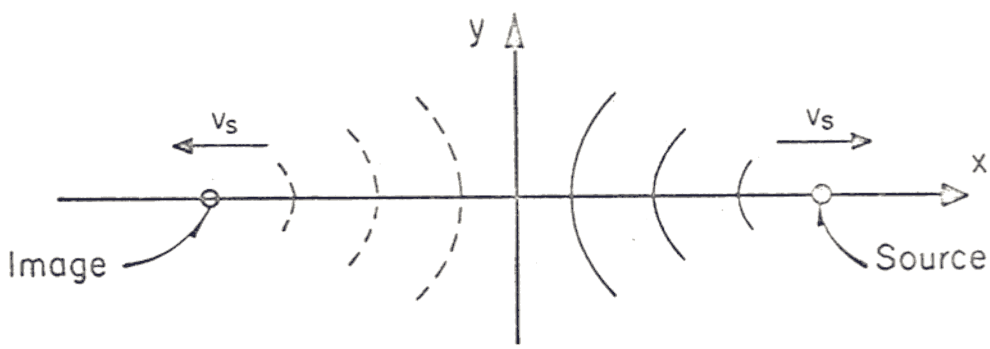


Fig 3 Source and image to satisfy the rigid dam condition

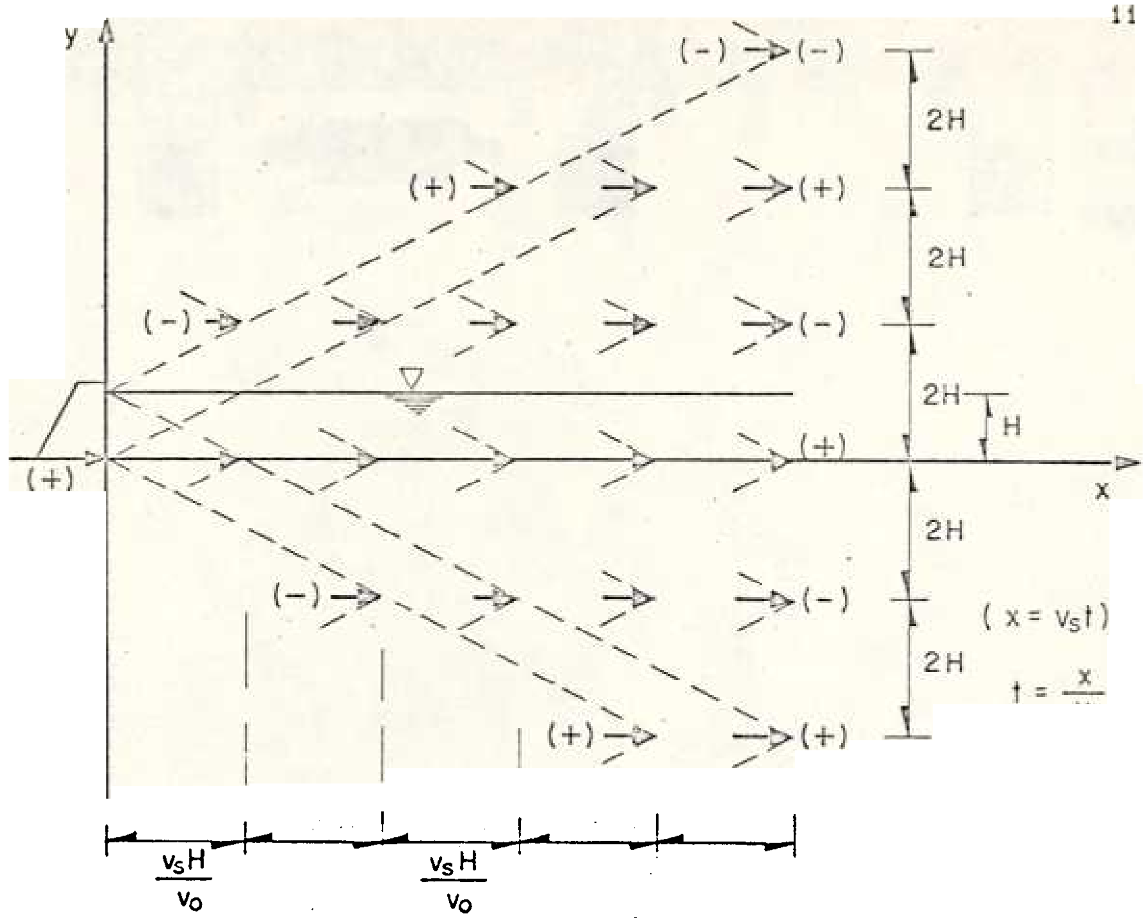


Fig 4 Source and images to satisfy water level and bottom conditions

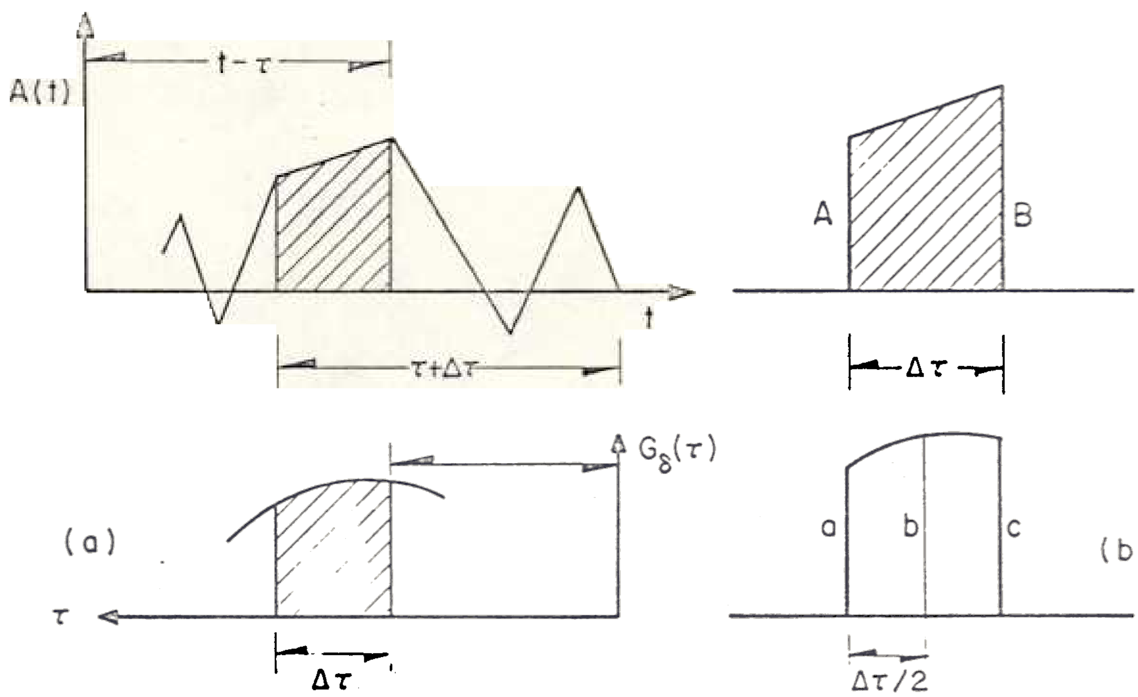


Fig 5 Computation of the effect of a segment of an earthquake

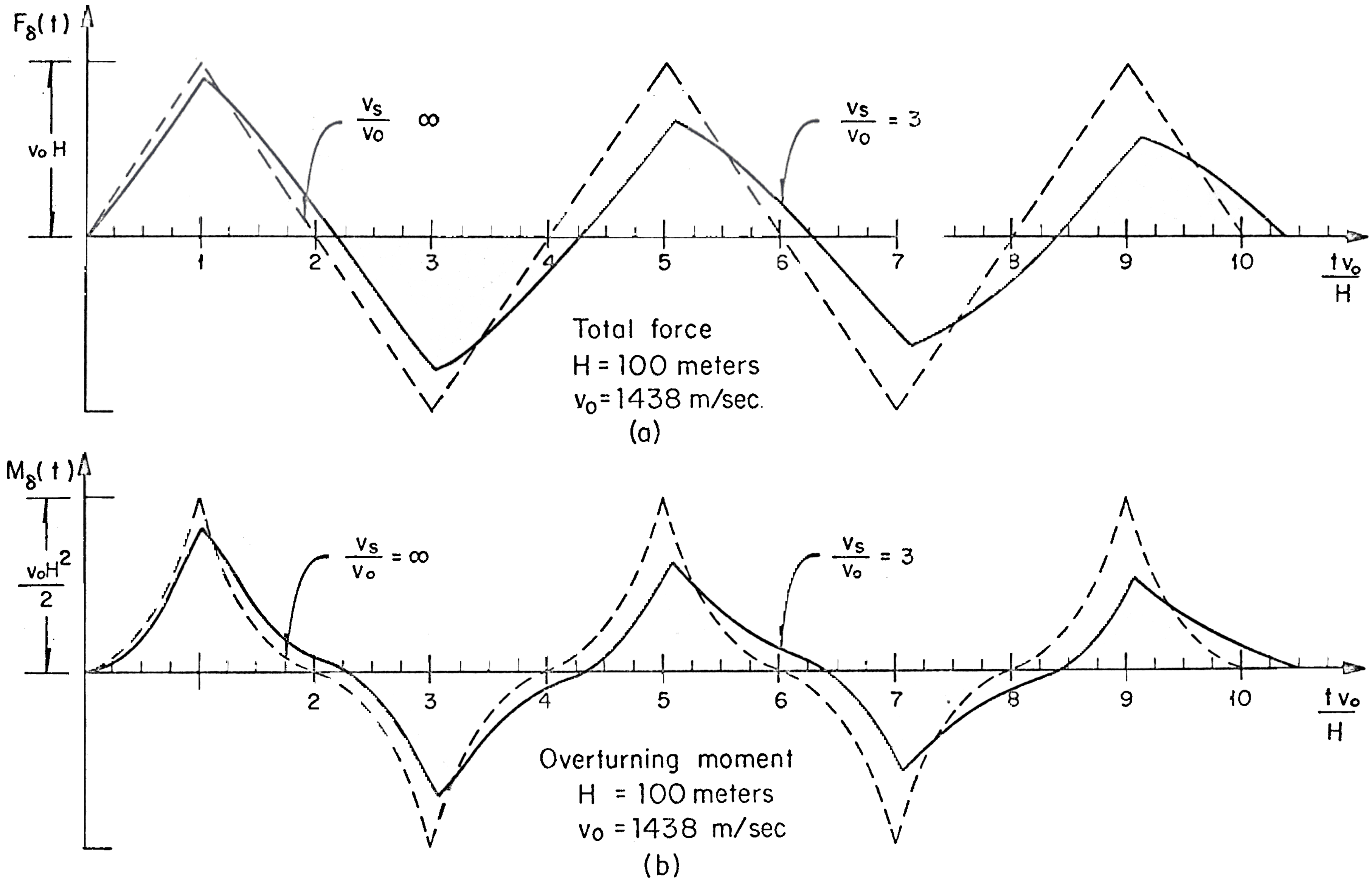


Fig 6 Hydrodynamic transfer functions for the upstream direction of the seismic waves (fig 1)

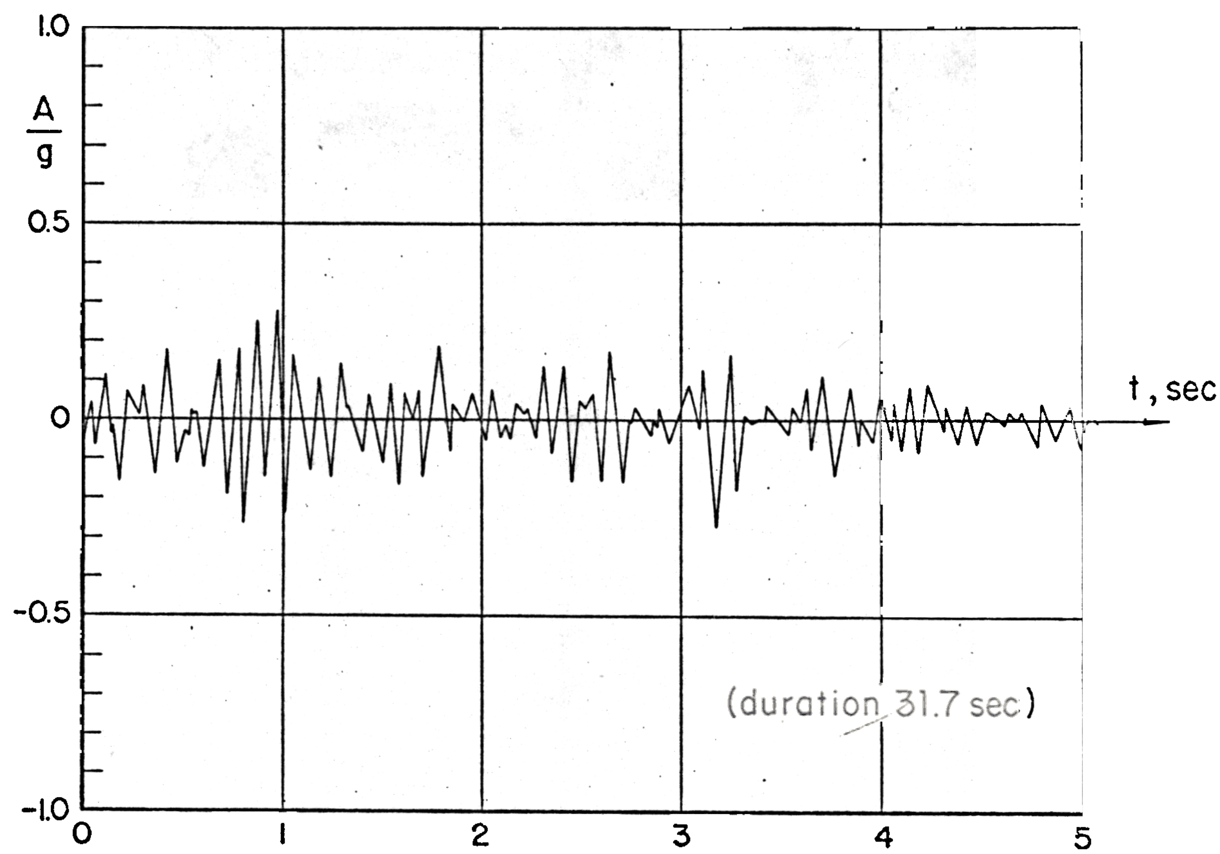


Fig 7 Vertical ground accelerations. E Centro, Cal, earthquake, May 18, 1940

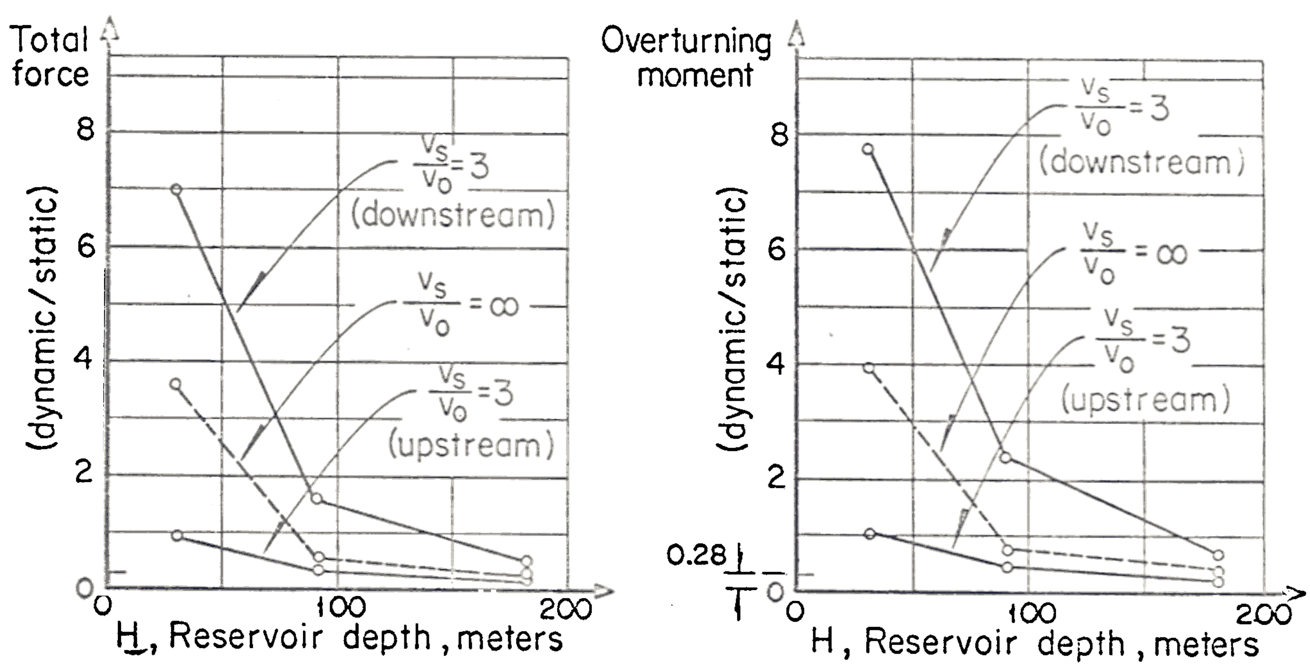


Fig 8 Maximum hydrodynamic response for traveling vertical accelerations of the El Centro earthquake

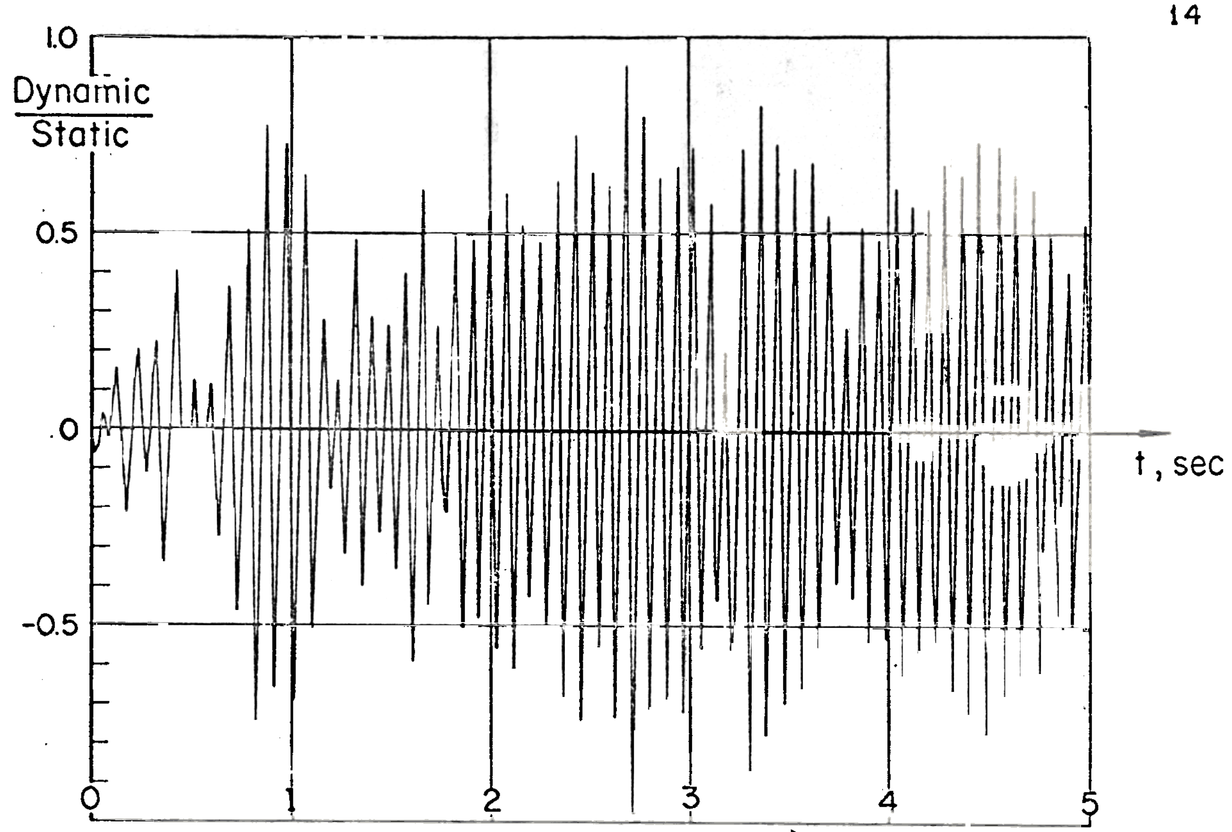


Fig 9 Total force on the dam. H=30.5 m (100'), $v_s/v_o = 3.0$
Earthquake traveling in the upstream direction

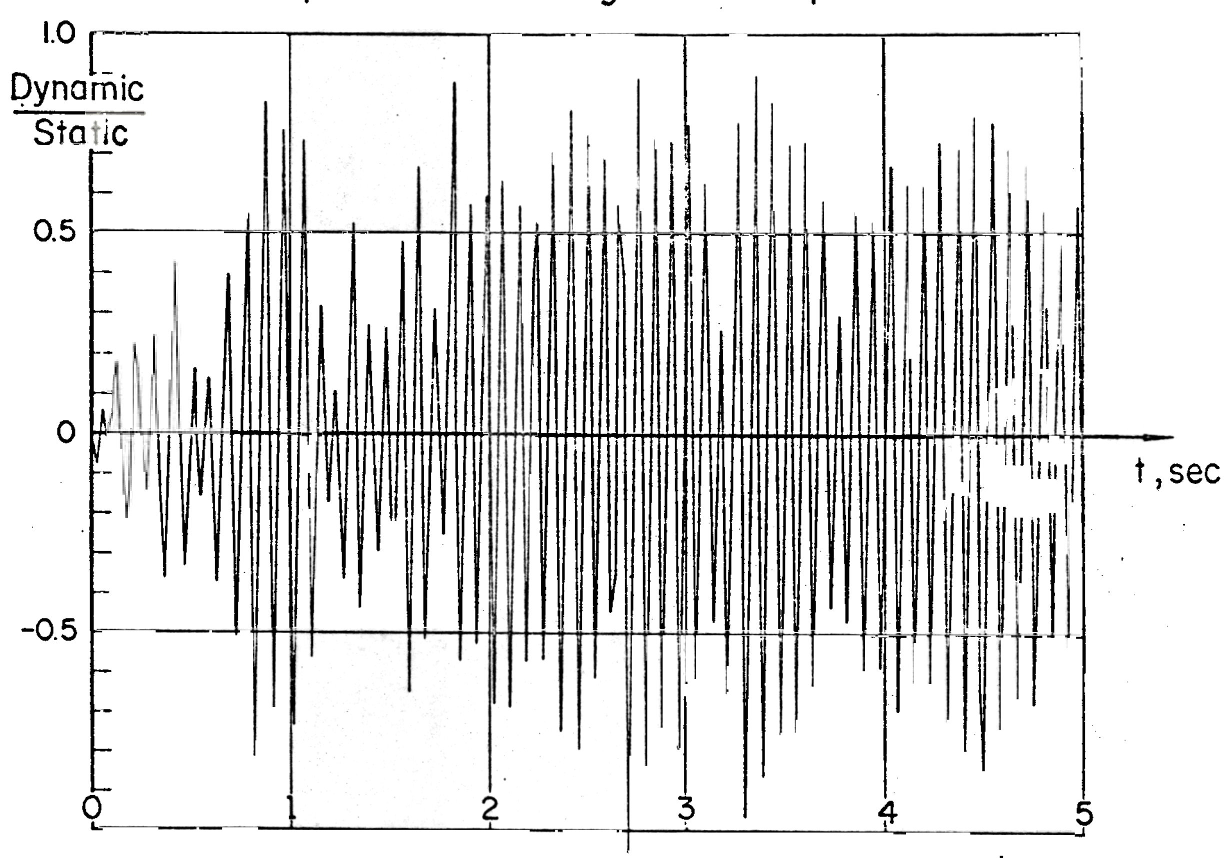
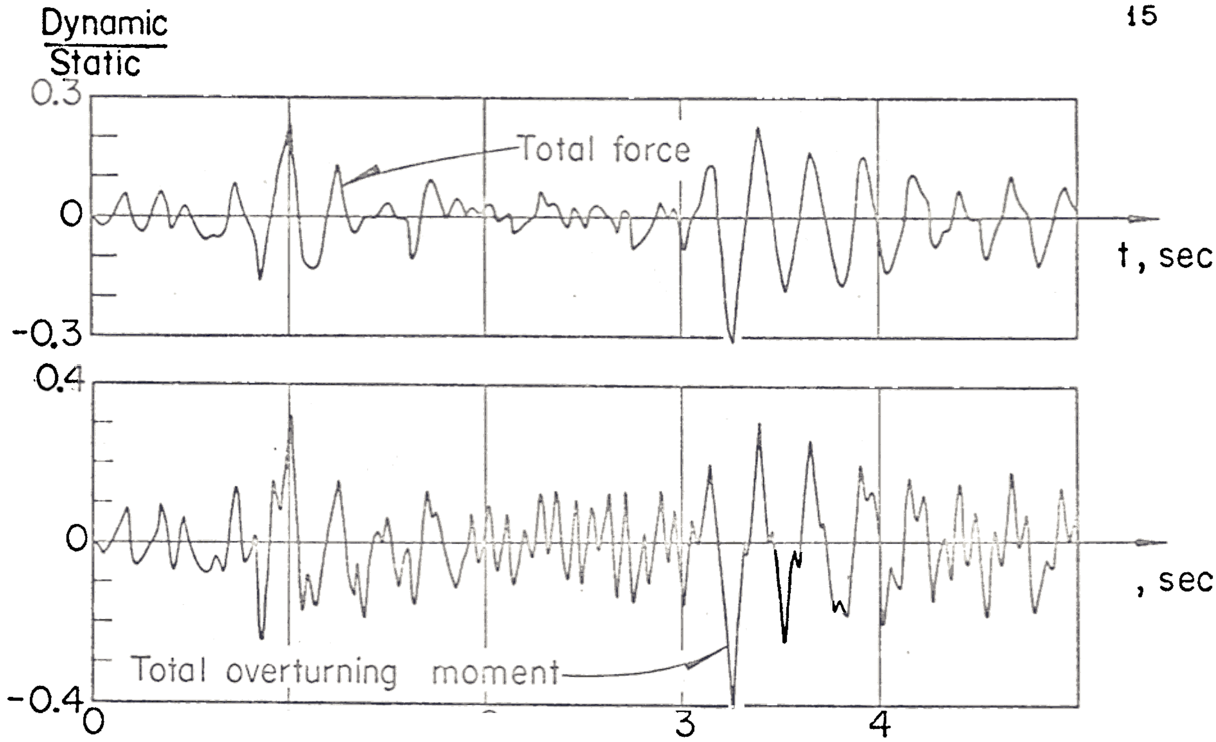
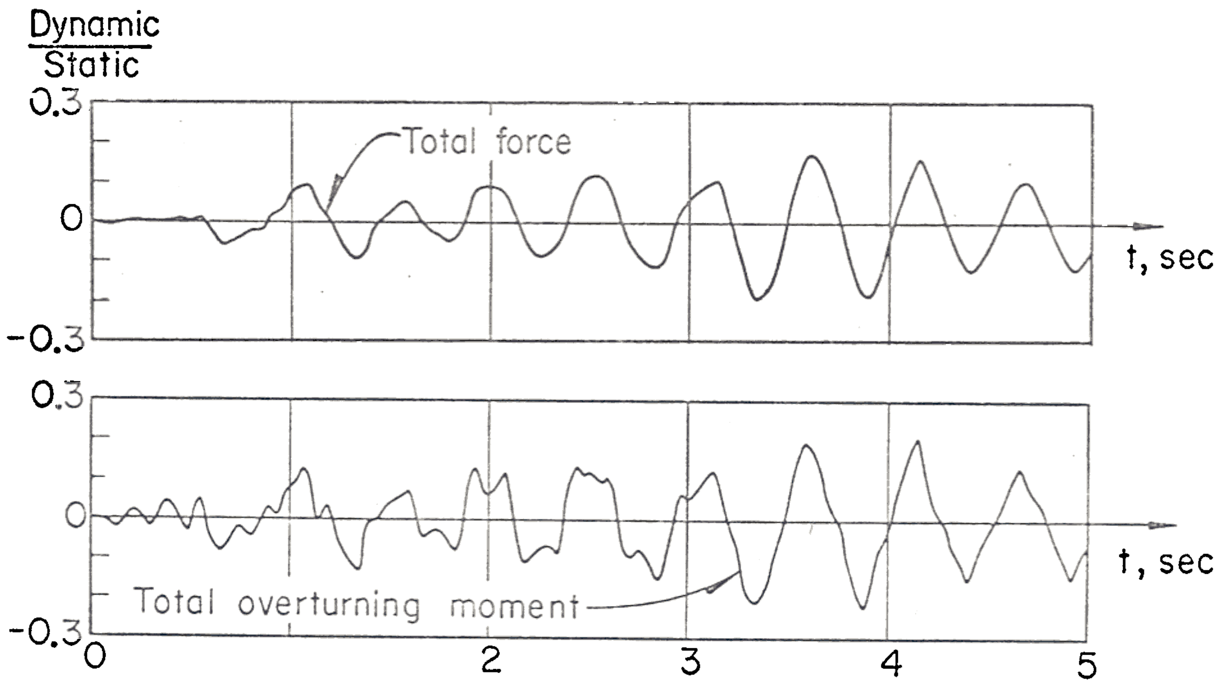


Fig 10 Total overturning moment. H=30.5m (100'), $v_s/v_o=3.0$
Earthquake in the upstream direction



Total force and overturning moment. H=91.4 m, (300'), V_s/V_0=3.0. Earthquake in the upstream direction



12 Total force and overturning moment. H= 82.9 m, (600'), V_s/V_o = 5.0. Earthquake in the upstream direction

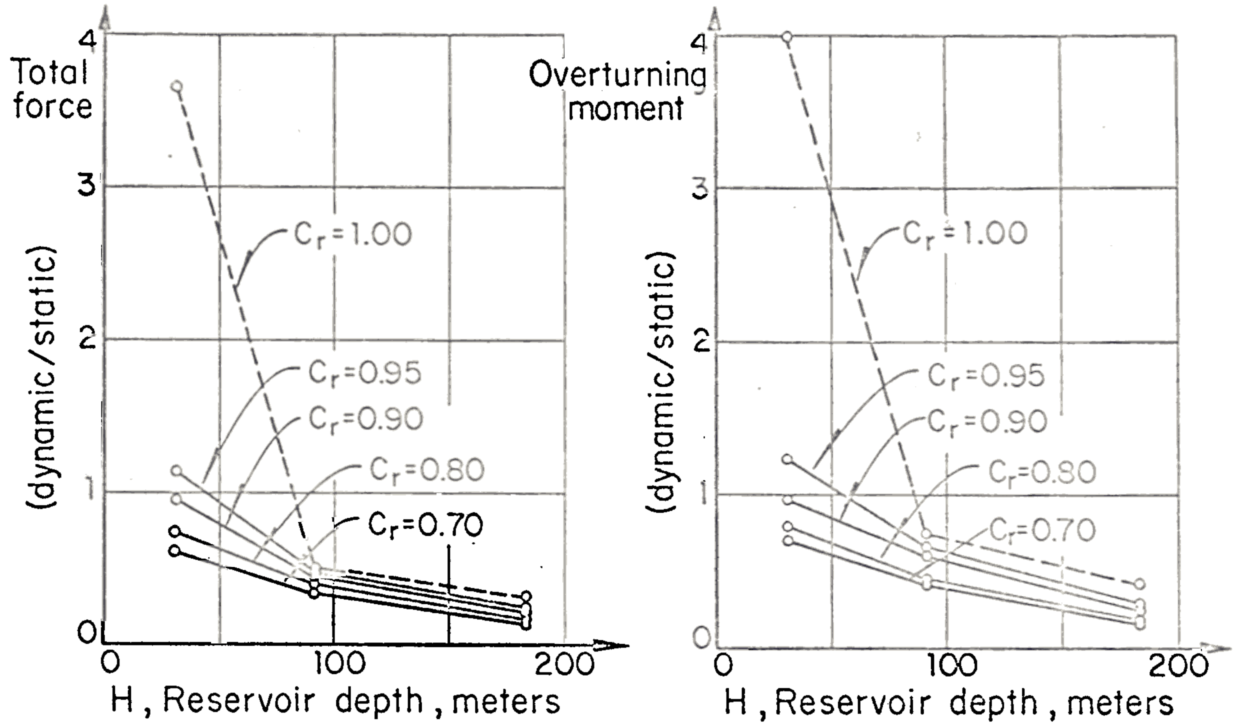


Fig 13 Maximum hydrodynamic response for uniform vertical motion of the E Centro earthquake

Table 1

Reservoir depth meters	Reservoir depth foot	$\frac{V_s}{V_0}$	Maximum total force dynamic/static (sec)	Maximum overturning moment dynamic/static (sec)
30.5	100	3.0 (D) ∞ 3.0 (U)	−7.05 (7.268) 3.63 (7.674) −0.98 (2.718)	−7.73 (7.268) 3.97 (7.847) −1.05 (2.718)
91.4	300	3.0 (D) ∞ 3.0 (U)	1.65 (7.420) −0.50 (7.849) −0.33 (3.253)	2.375(7.420) 0.76 (7.974) −0.42 (3.270)
182.9	600	3.0 (D) ∞ 3.0 (U)	−0.50 (2.267) −0.32 (3.848) −0.19 (3.343)	−0.64 (7.475) 0.42 (4.132) −0.22 (3.890)

(D) downstream direction of the earthquake
(U) upstream direction of the earthquake ,(fig 1)