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SOIL-STRUCTURE INTERACTION AS A DIFFRACTION PROBLEM

by

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SYNOPSIS

A mathematical formulation is presented for the problem of dynamic soil-structure interaction considering linear behavior of the soil material and arbitrary nonlinear structural properties. Using the unperturbed soil motion as a point of departure, the problem is formulated in a fashion similar to that for diffraction. Conditions for traction and displacement jumps are established that define a minimal set of data required to determine the resulting motion of the structure.

INTRODUCTION

Considerable attention has been devoted in recent years to the investigation of various aspects of dynamic soil-structure interaction [4-8], since it has been recognized that compliance of the soil foundation can be an important factor in design of earthquake resistant structures. In dealing with this problem it is customary to assume that the ground motion is known in the absence of the structure; it is then required to evaluate the structural response and perturbed motion of the soil with the unperturbed ground motion as excitation. The information about the excitation usually is supplied in the form of seismic records obtained at one or several locations and it is not sufficient to determine completely the characteristics of the incoming seismic waves; the direction of propagation of the waves, for instance, is not determined by a single seismic record. Although the available information has to be extrapolated in almost every application one can think of, knowledge of the unperturbed motion in the entire soil medium is not required. Precisely what information is essential to compute the structural response is, however, not clear. Considering the scarcity of available data, it is important to establish what are the minimum data required for this problem; this would allow carrying out the necessary extrapolation of available information in a more efficient manner.

In the present paper a formulation of the problem of dynamic soil-structure interaction is presented on the assumption that the ground motion is known in the absence of the structure. Under this assumption the problem consists in determining the effect that a region with different mechanical properties (i.e., the structure) has on the overall motion. Posed in this way, the problem appears as one of diffraction,

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for which the incident and reflected waves make up the unperturbed soil motion and the diffracted wave is the motion that needs to be added to the former to obtain the actual motion.

Diffraction problems have been treated extensively in seismology and the method to formulate them though used primarily for linear systems [1-3], is equally applicable to nonlinear ones. In this work the problem is formulated for an arbitrary nonlinear structure, assuming linear soil behavior. This formulation can also be applied if the soil behaves non-linearly within a bounded domain, provided the region representing the structure is enlarged sufficiently to enclose the nonlinear region of the soil.

It is assumed that at the construction site the excavation for the foundation is not necessarily filled completely by the structure (fig. 1) and it is shown that a minimal set of data required for the treatment of the problem is made of the displacements that the base of the excavation would have if structure and excavation were not present. A systematic procedure is established to incorporate this set of minimal data into the formulation of this class of problems; it appears as prescribed jump conditions that supplement the usual set of partial differential equations, initial and boundary conditions. The resulting system is amenable to solution by the finite element method or some other numerical technique.

THE DIFFRACTION PROBLEM

Let $\hat{R}$ be the region occupied by the soil before the excavation and structure were built, and $\partial\hat{R}$ the boundary of $\hat{R}$. In most applications $\hat{R}$ will be a half-space and $\partial\hat{R}$ the free surface of the soil; this case is illustrated in Fig. 1.

It will be assumed that the building or structure is made up of an excavation E and a structure S; the region occupied by both the excavation and the structure will be called the region of construction C. Two parts of the excavation E will be distinguished, the excavation occupied by the structure, denoted by E_S and the part of the excavation that remains free, denoted by E_F. This way of formulating the problem includes the case when there is no excavation; i.e. when E is empty (Fig. 2). The region occupied by the soil after building the excavation and the structure will be denoted by R. The boundaries of E,S,C and R will be denoted by ∂E, ∂S, ∂C and ∂R respectively. On the other hand, $\partial_F R$ and $\partial_S R$ will be the parts of the boundary of R that are shared with E_F and E_S respectively.

Let $\hat{\underset{\sim}{u}}(\underset{\sim}{x},\underset{\sim}{t})$ be the motion due to an earthquake; it will be assumed that $\hat{\underset{\sim}{u}}$ defined on $\hat{R}$ for every $t \geq 0$, is such that at $t=0$ the region of construction C has not been perturbed; i.e. initial displacements and velocities vanish in this region. The soil will be assumed linear elastic and therefore, $\hat{u}_i$ and the corresponding stresses $\hat{\tau}_{ij}$ satisfy in $\hat{R}$ the linear equations of motion and the usual linear constitutive relation between stress and deformation tensors, along with a traction free boundary condition on $\partial\hat{R}$.

The unperturbed motion $\hat{u}_i$ of the earthquake will be modified when a structure is built on the soil. The problem of soil-structure interaction consists in finding a motion $u^T(x,t)$ defined on $R \cup S$ for $t \geq 0$, such that its initial displacements and velocities coincide with those of the motion $\hat{u}$ on R, and vanish on the structure because it has not been excited at t=0. Of course, u^T is required to satisfy in addition the same equations of motion and constitutive relations on R, as u, along with the traction free boundary condition on ∂R-S. The properties of the structure are arbitrary and therefore, the field equations of motion satisfied by the displacements $u_i^T(x,t)$ at the structure will be left unspecified. Irrespectively of these, displacements and tractions must be continuous across the surface that separates the soil from the structure; thus,

$$[u_i^T] = 0 \qquad \text{on } \partial_S R \qquad (1.a)$$

$$[\tau_{ij}^T]n_j = 0 \qquad \text{on } \partial_S R \qquad (1.b)$$

Here, brackets refer to the jump of the function contained within; i.e. the value of the function on the positive side minus its value on the negative one. The unit normal vector n_i is directed outwards from the soil (i.e. from R) and therefore the positive side faces the structure, while the negative one, the soil.

If the structure consists of an inviscid fluid, as when studying the seismic response of a dam, equations (1) are not satisfied, because in such case only the normal components of tractions and displacements are continuous. The modifications required in the above formulation to include this case are straight-forward, but will not be included here for the sake of brevity.

The procedure that will be used to formulate soil-structure interaction as a problem of diffraction is similar to that used previously by one of the authors [1-3]. The displacements $u(x,t)$ on $R \cup S$ and $t \geq 0$, are defined by

$$u_i = \begin{cases} u_i^T - \hat{u}_i & \text{on } R \\ u_i^T & \text{on } S \end{cases} \qquad (2)$$

The stresses associated to u_i will be denoted by τ_{ij}.

Using this definition, it follows that

$$\frac{\partial \tau_{ij}}{\partial x_j} - \rho \frac{\partial^2 u_i}{\partial t^2} = 0; \; \tau_{ij} = \hat{C}_{ijpq} \frac{\partial u_p}{\partial x_q} \quad \text{on } R \qquad (3)$$

$$u_i(\underset{\sim}{x},0) = \frac{\partial u_i}{\partial t}(\underset{\sim}{x},0) = 0 \qquad \text{on } R \qquad (4.a)$$

$$\tau_{ij}\, n_j = 0 \qquad \text{on } \partial R - S \qquad (4.b)$$

$$\tau_{ij}\, n_j = -\hat{\tau}_{ij}\, n_j \qquad \text{on } \partial_F R \qquad (4.c)$$

$$[u_i] = \hat{u}_i \qquad \text{on } \partial_S R \qquad (5.a)$$

$$[\tau_{ij}] n_j = \hat{\tau}_{ij}\, n_j \qquad \text{on } \partial_S R \qquad (5.b)$$

In addition, the displacements $\underset{\sim}{u}$ which are identical with the total displacements on the structure S, satisfy there the equations of motion for the structure and suitable conditions at t=0 and on its free boundary $\partial S - R$. These conditions together with equations 3 to 5 define a well posed problem for $\underset{\sim}{u}(\underset{\sim}{x},t)$ on $R \cup S$, assuming a very general kind of behavior for the structure, which includes the possibility that it be made of non-linear materials with memory. As mentioned previously, if part of the structure consists of an inviscid liquid (as when dams are analyzed), conditions 1 are not fulfilled and consequently, equations 5 must be modified.

MINIMAL INFORMATION TO STUDY INTERACTION

The formulation presented in the last section has implications worth discussing. When studying soil-structure interaction problems, it is frequently not clear which are the earthquake data that are essential for the analysis of the structural response. Eqs. 3 to 5 supply a precise answer to this question. Indeed, the total motion in the structure is given by $\underset{\sim}{u}(\underset{\sim}{x},t)$, which is determined through the problem formulated in the previous section. The only data appearing in the system 3 to 5 are the displacements $\hat{u}_i$ and tractions $\hat{\tau}_{ij} n_j$ associated with the motion of the unaltered ground, that take place where the bottom of the construction is to be built. Thus, when analyzing the problem of interaction at a site, it is necessary to estimate, either by direct observation or by some other means, the displacements and tractions that an earthquake would produce at the bottom of the construction, but any additional information is irrelevant to study the behavior of the structure.

This information can be reduced even further, because tractions and displacements are not independent at the bottom of the excavation ($R \cap \partial E$). Indeed, define for $t \geq 0$

$$U_i(\underset{\sim}{x},t) = \hat{u}_i(\underset{\sim}{x},t);\; T_i(\underset{\sim}{x},t) = \hat{\tau}_{ij}(\underset{\sim}{x},t) n_j \quad \text{on } R \cap \partial C \qquad (6)$$

Then the displacements $\hat{u}_i$ and tractions $\hat{\tau}_{ij}$ satisfy in the region of the excavation $E \subset \hat{R}$, the linear equations of motion, the usual linear constitutive relations between stress and deformation tensors and, conditions of initially vanishing displacements and velocities. These, together with the boundary conditions implied by the fact that $E \cap \partial\hat{R} \subset \partial\hat{R}$ is traction free and $\hat{u}_i$ is given on $R \cap \partial C$ by the first of Eqs. 6, define a well posed problem for $\hat{u}_i(\underset{\sim}{x},t)$ which determines it on E for $t \geq 0$ when $\underset{\sim}{U}_i(\underset{\sim}{x},t)$ is prescribed. Once $\hat{u}_i(\underset{\sim}{x},t)$ has been obtained on E, $\hat{\tau}_{ij}$ is determined by the constitutive relation of the soil and $T_i(\underset{\sim}{x},t)$ by the second of eqs. 6. The preceding discussion shows that a minimal set of data for the problem of interaction are the displacements that the earthquake would produce at the base of the construction if the ground had not been yet altered. It can be shown that this result remains valid when there is no excavation (Fig. 2).

A systematic procedure to incorporate the minimal set of data is by means of eqs. 3 to 5. However, it is required to express previously the traction $\underset{\sim}{T}$ in terms of the displacement data $\underset{\sim}{U}$; this can be done solving on E the initial-boundary value problem which determines $\hat{\underset{\sim}{u}}$ in terms of $\underset{\sim}{U}$.

THE LINEAR STRUCTURE

As an illustration the theory presented herein will be applied to a case for which the structure S consists of a linear elastic material. To keep the notation simple, it is convenient to define the elastic tensor $C_{ijpq}(\underset{\sim}{x})$ on $R \cup S$, so that it corresponds to the structure or the soil depending on whether $\underset{\sim}{x} \varepsilon S$ or $\underset{\sim}{x} \varepsilon R$; thus

$$C_{ijpq}(\underset{\sim}{x}) = \hat{C}_{ijpq}(\underset{\sim}{x}) \quad \text{on } R \tag{7}$$

In this case the soil-structure interaction is governed by the system of equations 3 to 5, except that eqs. 3 have to be replaced by

$$\frac{\partial \tau_{ij}}{\partial x_j} - \rho \frac{\partial^2 u_i}{\partial t^2} = 0 \; ; \; \tau_{ij} = C_{ijpq} \frac{\partial u_p}{\partial x_q} \quad \text{on } R \cup S \tag{8}$$

while eq. (4.a) holds on $R \cup S$ and eq. (4.b) holds on $\partial(R \cup S) - \partial_F E$.

A minimal set of data for this problem are the displacements $U_i = \hat{u}_i$ on $R \cap \partial C$. To use it, however, it is necessary to supplement the system of equations just mentioned, with the system that determines the tractions $\underset{\sim}{T}$ in terms of the displacement data $\underset{\sim}{U}$. An alternative would be to prescribe $\underset{\sim}{U}$ and $\underset{\sim}{T}$ separately, but when doing so it is required to make sure that they are compatible, because otherwise they would lead to absurd results.

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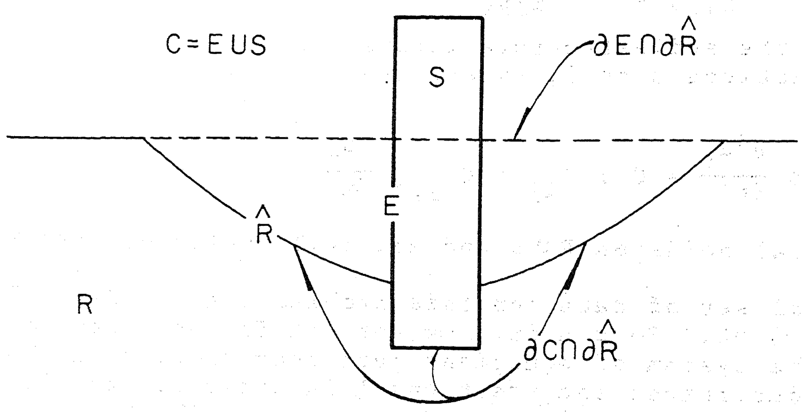

Fig 1. Soil-structure system with excavation

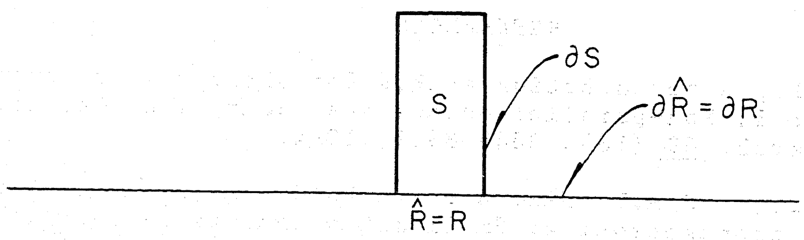

Fig 2. Structure supported on the soil surface