Reply

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Before going into the subject matter of the discussion, some remarks are in order. The purpose of our paper was not to develop a solution to the problem of linearized free surface flow to a pumping well, and we did not develop such a solution. As we mentioned there, exact solutions for that problem are available [*Boulton*, 1954; *Neuman*, 1972]; however, by an omission that we regret, we did not refer to *Dagan*'s [1967] solution. We recall these facts because, in his comment, Dagan refers to 'the solution by *Herrera et al.* [1978].' As mentioned above, we did not construct or propose any solution.

We feel that this apparent confusion requires that we make more precise the scope of our work, and we want to thank Dagan because his comments have given us an opportunity to clarify it.

In our paper we prove that when the linearized theory of free surface flow is supplemented with an additional hypothesis (equation (15)), the flow is governed by an integrodifferential equation which reduces to Boulton's equation in the incompressible case. Thus the final product is an integrodifferential equation, not a solution. As we stated in the introduction, 'probably the main interest of the results presented in this paper is theoretical.' We think so, because the systematic derivation we have obtained of Boulton's integrodifferential equation, starting from basic principles and using an additional well-defined hypothesis, goes beyond previous attempts that had been made [*Streltsova*, 1972, 1973]. Such derivation contributes to the understanding of the physical process of delayed yield response. It also clarifies some of the problems that a previous paper by *Gambolati* [1976] addressed. namely, the physical meaning of Boulton's α.

We think that is the main interest of our results. In particular, we do not feel that our theory in its present state of development is suitable to be applied directly to the solution of specific problems. Our reasons are essentially those stated by Dagan in his comments.

However, by further developing the theory, Herrera has shown that corresponding to the series expansion that we give for the kernel, there is a series expansion that can be used to obtain the solution for a partially penetrating well and to develop an efficient numerical method for the treatment of free aquifers. This, however, will be the subject matter of another paper that is now being prepared.

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(Received December 1, 1978;
accepted December 4, 1978.)

Paper number 8W1311.
0043-1397/79/008W-1311$01.00

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Paper number 8W1311.
0043-1397/79/008W-1311$01.00