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SURFACE MOTION OF TOPOGRAPHICAL IRREGULARITIES FOR INCIDENT *P*, *SV*, AND RAYLEIGH WAVES

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ABSTRACT

Trefftz's method is applied to solve the diffraction of *P*, *SV*, and Rayleigh waves by surface irregularities in an elastic, isotropic and homogeneous half-space. The diffracted wave fields are constructed with linear combinations of solutions which form *c*-complete or *T*-complete (*T* after Trefftz) families for the wave equation. Boundary conditions are satisfied in the least-squares sense on the irregularity and on a finite interval of the half-space surface. Excellent agreement was found with the solution by Wong (1982).

INTRODUCTION

A close relation has been suggested between earthquake damage and topographic and geological irregularities (Poceski, 1969; Hudson, 1972). The influence of local conditions is relevant in seismic risk assessment (Esteva, 1977) and in calculating the response of long structures (Esquivel and Sánchez-Sesma, 1980).

Many research papers on the subject have appeared. For *SH*-wave diffraction, the problem has been dealt with using analytical solutions (e.g., Trifunac, 1971; Wong and Trifunac, 1974), integral equations (e.g., Wong and Jennings, 1975), boundary methods (e.g., Sánchez-Sesma and Rosenblueth, 1979; England *et al.*, 1980; Sánchez-Sesma *et al.*, 1982a), and finite elements (e.g., Smith, 1975; Aranda and Ayala, 1978).

For incidence of *P*, *SV*, and Rayleigh waves, the solution is more complicated because of the coupling of boundary conditions. To solve this problem many techniques have been proposed. For small slope irregularities, the perturbation method (Herrera, 1964; Hudson, 1967; McIvor, 1969; Hudson and Boore, 1980) and asymptotic expansions have been used (Sabina and Willis, 1977). Under the assumption of periodicity, solutions have been obtained for different wave fields and scatterers (e.g., Aki and Larner, 1970; Bouchon, 1973; Bouchon and Aki, 1977; Bard and Bouchon, 1980; Bard, 1982). The finite difference method has also been applied to problems of wave scattering (e.g., Boore, 1972; Boore *et al.*, 1981; Harmsen and Harding, 1981). For a semi-spherical cavity and incidence of elastic *P* and *S* waves, a solution, which employs power series expansions of the involved wave fields, has been obtained (Lee, 1982).

A boundary method has been applied to solve cases of incidence of *P*, *SV*, and Rayleigh waves at canyon-like topographies (Sánchez-Sesma, 1978; Wong, 1982) and alluvial valleys (Dravinski, 1982). This method uses solutions of discrete line sources of *P* and *SV* waves placed outside the region of interest to construct the scattered fields. Boundary conditions are satisfied in the least-squares sense on the surface of the irregularity. The method can be considered as a generalized inverse one (Wong, 1982). However, the use of the line sources, calculated approximately from Lamb (1904) integrals can make impractical this approach because of the strong computational effort needed in some cases. On the other hand, the location of sources and their number requires great care and the use of iterative criteria.

In this work, a different approach is presented. We use Trefftz's method. In recent years, considerable progress has been made in the formulation and understanding of this method which has just been published in book form (Herrera, 1984). In a previous paper (Sánchez-Sesma *et al.*, 1982a), a brief but systematic exposition of the method was presented in connection with diffraction of *SH* waves. Here, the diffracted fields are constructed using, instead of Lamb sources, *c*-complete or *T*-complete (*T* after Trefftz) families of solutions of the wave equation (Herrera, 1984). Then, multi-pole expansions for the potentials are formed in terms of Hankel functions. Since each one of the members of this set does not satisfy in itself the free-boundary conditions on the half-space surface, the matching on the boundary should include also the free plane surface, but fortunately not all. This fact requires some comments. The results by Domínguez (1978) on the dynamic response of surface and embedded footings show the small influence of the free boundary; only a small part had to be discretized in order to get good results. Calculations for a dynamic contact problem (Sánchez-Sesma *et al.*, 1982b; Bravo and Sánchez-Sesma, 1985a) showed that convergent results can be obtained if the length of this zone is of, say, two or three times the characteristic dimension of the vibrating body.

The formulation of the problem is briefly presented, and numerical results are given for a semi-circular canyon under incidence of *P*, *SV*, and Rayleigh waves. Comparisons with results by Wong (1982) show excellent agreement. The solution with multi-pole sources is, computationally speaking (save for the increase in the

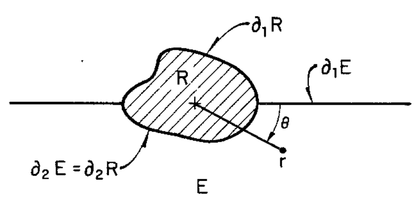

FIG. 1. Regions R, E, its boundaries, and cylindrical coordinates.

number of variables), similar to the solution of the scalar *SH* case (e.g., Sabina *et al.*, 1978; England *et al.*, 1980; Sánchez-Sesma *et al.*, 1982a). Then, the cost is greatly reduced and the formulation of the problem becomes very simple. Moreover, general theory (Herrera, 1984) shows that the property of *T*-completeness is independent of the particular shape of the region considered.

This approach has been applied to study the response of two-dimensional alluvial valleys (Bravo and Sánchez-Sesma, 1985b) and, using an azimuthal decomposition, to study diffraction of elastic waves by three-dimensional surface irregularities (Sánchez-Sesma, 1983; Sánchez-Sesma *et al.*, 1984).

Formulation of the problem

Consider the elastic half-space and a two-dimensional surface irregularity, which, in Figure 1, are denoted by E and R, respectively. Let $\partial_1 E$ and $\partial_1 R$ be the free boundaries of the regions, and $\partial_2 E = \partial_2 R$ be the common boundary between them. Under incidence of elastic waves, the total field is obtained by superposition of diffracted waves on the *free-field* solution, i.e., on the solution in absence of irregularity.

For harmonic dependence of time given by the factor $\exp(i\omega t)$, the displacement vector $\bar{u}$, must satisfy the reduced Navier equation

$$\mu\nabla^2 \bar{u} + (\lambda + \mu)\, \nabla\nabla\cdot\bar{u} + \rho\omega^2\bar{u} = 0 \tag{1}$$

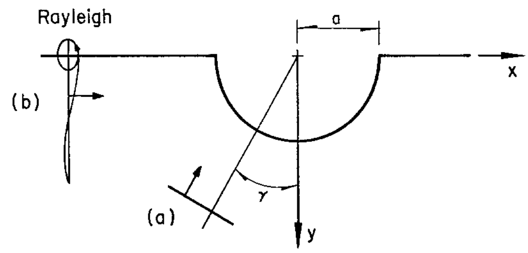

FIG. 2. Semi-circular canyon of radius a and incident plane waves. (a) Homogeneous plane wave with incidence angle γ. (b) Rayleigh surface wave.

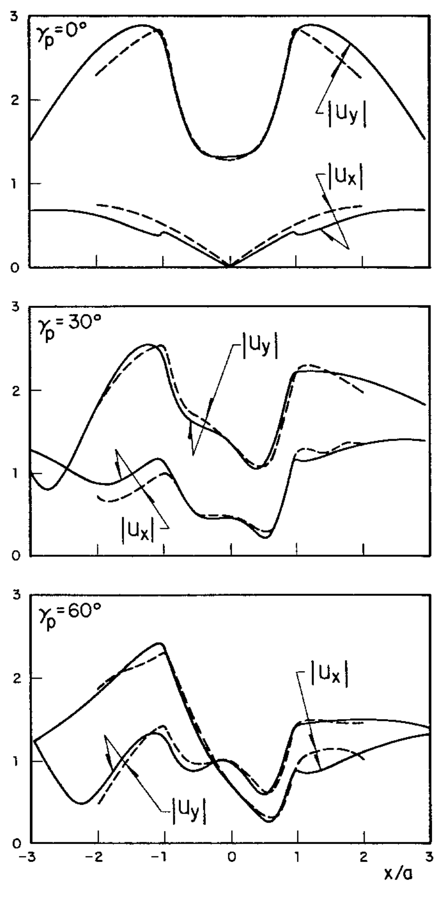

FIG. 3. Amplitudes of the horizontal (u_x) and vertical (u_y) displacements for incidence of P waves upon a semi-circular canyon. Normalized frequency $\eta_k = 0.5$ and incidence angles $\gamma_p = 0°$, 30°, and 60°. Comparison of results with the ones obtained by Wong (1982).

where λ, μ = Lamé constants, ρ = mass density, and ω = circular frequency. The elastic constants and the density should be particularized for each medium.

Boundary conditions are those of zero tractions at $\partial_1 R$ and $\partial_1 E$ and continuity of displacements and tractions across $\partial_2 R = \partial_2 E$. In addition, the diffracted fields must satisfy the Sommerfeld-Kupradze elastic radiation condition at infinity (Sommerfeld, 1949; Kupradze, 1965).

Method of Solution

Let us write the total fields in the form

$$\bar{u}^E = \bar{u}^{(0)} + \sum_{n=-N}^{N} A_n \bar{w}_n{}^E(P) + \sum_{n=-N}^{N} B_n \bar{w}_n{}^E(SV) \tag{2}$$

for the region E, and

$$\bar{u}^R = \sum_{m=-M}^{M} C_m \bar{w}_m{}^R(P) + \sum_{m=-M}^{M} D_m \bar{w}_m{}^R(SV) \tag{3}$$

for the region R. In equation (2), $\bar{u}^{(0)}$ = displacement vector of the free-field solution, $\bar{w}_n{}^E(P)$ and $\bar{w}_n{}^E(SV)$ are the displacement vectors of the P and SV scattered fields, respectively. In equation (3), $\bar{w}_m{}^R(P)$ and $\bar{w}_m{}^R(SV)$ are the displacement vectors of the P and SV refracted fields, respectively. A_n, B_n, C_m, and D_m are unknown coefficients to be determined from boundary conditions; and N, M are the order of the approximations.

Let us write the two types of fields of solutions, P and SV body waves, as potentials of the form

$$\phi_E(P) = \sum_{n=-N}^{N} A_n H_n^{(2)}(qr) e^{\mathrm{i}n\theta} \tag{4a}$$

$$\psi_E(SV) = \sum_{n=-N}^{N} B_n H_n^{(2)}(kr) e^{\mathrm{i}n\theta} \tag{4b}$$

for the region E, and

$$\phi_R(P) = \sum_{m=-M}^{M} C_m J_m(qr) e^{\mathrm{i}m\theta} \tag{5a}$$

$$\psi_R(SV) = \sum_{m=-M}^{M} D_m J_m(kr) e^{\mathrm{i}m\theta} \tag{5b}$$

for the region R. Here, $q = \omega/\alpha = P$ wavenumber, $k = \omega/\beta = S$ wavenumber, $\alpha = \sqrt{(\lambda + 2\mu)/\rho} = P$-wave velocity, $\beta = \sqrt{\mu/\rho} = S$-wave velocity, and r, θ = cylindrical coordinates. In equations (4) and (5), $H_n^{(2)}(\cdot)$ = Hankel function of the second kind and order n, and $J_m(\cdot)$ = Bessel function of the first kind and order m, respectively. Expressions for the associated displacement and stress fields can be found in the literature (e.g., Mow and Pao, 1971).

By imposing boundary conditions at a finite number of points on the boundaries, a system of linear equations for the coefficients is obtained, in which the independent part is given in terms of the free-field solution. Because of each one of the solutions which appear in equations (5) does not satisfy in itself the boundary conditions in $\partial_1 E$, the numerical treatment must be extended to part of the half-space surface. It is convenient to form an overdetermined linear system and solve it in the least-squares sense (Sánchez-Sesma and Rosenblueth, 1979; Wong, 1982). Once the coefficients are known, equations (2) and (3) allow for the calculation of displacement fields.

We could include two additional terms in equations (4) and (5) to consider the diffracted Rayleigh waves. In that case, the treatment of boundary conditions should be slightly modified. However, as we are interested in the surface displacement amplitudes on and near the irregularity, the diffracted fields are constructed only with body waves. This fact may introduce some errors in the amplitudes and phases of the displacement fields. Current research on the subject is being performed.

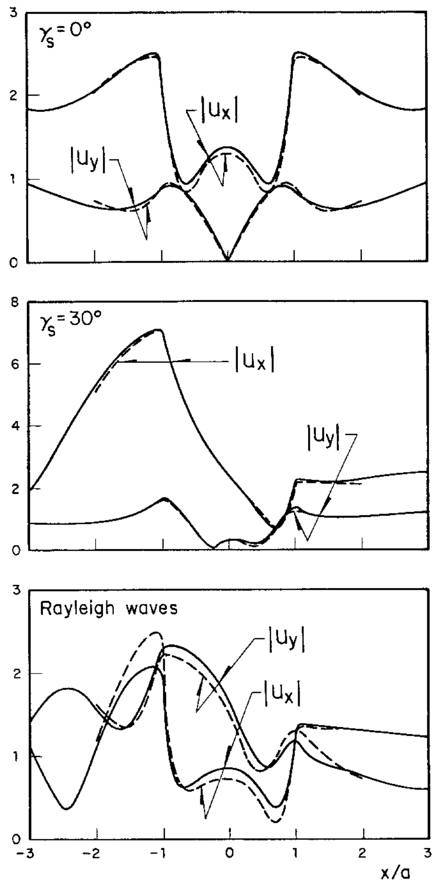

FIG. 4. Amplitudes of the horizontal (u_x) and vertical (u_y) displacements for incidence of SV and Rayleigh waves upon a semi-circular canyon. Normalized frequency $\eta_k = 0.5$ and incidence angles $\gamma_s =$ 0° and 30° for SV waves. Comparison of results with the ones obtained by Wong (1982).

Numerical Results

In order to show the performance of the method, the surface displacements were calculated on the boundary of a semi-circular canyon for incidence of P, SV, and Rayleigh waves (see Figure 2). All results are given for a Poisson ratio of 1/3, and a normalized frequency $\eta_k = \omega a/\pi\beta = 0.5$, where a = radius of the canyon.

The orders of the expansions and the number and location of the collocation points are obtained using a "trial and error" procedure which is based upon the error analysis of boundary conditions and the stability of the surface displacement field. Further research is needed for practical assessment of the calculation parameters. The results presented here were obtained with an order of the exterior expansions of 8 and a total of 60 collocation points, uniformly placed along $\partial_2 E$ and

$\partial_1 E$ in a length of three radii at each side of the canyon; they are displayed in Figures 3 and 4. In all cases, comparison is provided with Wong's (1982) method. The agreement is excellent. The largest differences appear for the case of incident Rayleigh waves. A still better agreement could be expected if the diffracted Rayleigh waves were included in the analysis.

Conclusions

Some results are presented for the surface displacements generated by incidence of P, SV, and Rayleigh waves upon a semi-circular canyon on the surface of an elastic half-space. These results were obtained by means of Trefftz method (Herrera, 1984) using T-complete multi-pole expansions for the displacement potentials. Boundary conditions on the irregularity itself and on a part of the half-space surface are satisfied in the least-squares sense.

An excellent agreement with the solution by Wong (1982) is found. This fact and the simplicity of the procedure presented confirm, as we have indicated previously (Sánchez-Sesma *et al.*, 1982a), that the method can be used with advantages in many problems of earthquake engineering and seismology.

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References

Aki, K. and K. L. Larner (1970). Surface motion of a layered medium having an irregular interface due to incident plane *SH* waves, *J. Geophys. Res.* **75,** 933–954.

Aranda, G. R. and G. A. Ayala (1978). Modelo numérico eficiente de aplicación en estudios de amplificación dinamica, Instituto de Ingeniería, UNAM, México.

Bard, P. Y. (1982). Diffracted waves and displacement field over two-dimensional elevated topographies, *Geophys. J. R. Astr. Soc.* **71,** 731–760.

Bard, P. Y. and M. Bouchon (1980). The seismic response of sediment filled valleys. Part 1. The case of incident *SH* waves, *Bull. Seism. Soc. Am.* **70,** 1263–1286.

Boore, D. M. (1972). A note on the effect of simple topography on seismic *SH* waves, *Bull. Seism. Soc. Am.* **62,** 275–284.

Boore, D. M., S. C. Harmsen, and S. T. Harding (1981). Wave scattering from a steep change in surface topography, *Bull. Seism. Soc. Am.* **71,** 117–125.

Bouchon, M. (1973). Effect of topography on surface motion, *Bull. Seism. Soc. Am.* **63,** 615–632.

Bouchon, M. and K. Aki (1977). Near-field of a seismic source in a layered medium with irregular interfaces, *Geophys. J. R. Astr. Soc.* **50,** 669–684.

Bravo, M. A. and F. J. Sánchez-Sesma (1985a). Vertical vibration of a rigid plate. Solution using Trefftz's method (submitted for publication).

Bravo, M. A. and F. J. Sánchez-Sesma (1985b). Ground motion on alluvial deposits under incidence of *P*, *SV* and Rayleigh waves (submitted for publication).

Domínguez, J. (1978). Dynamic stiffness of rectangular foundations, Report R-78-20, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Dravinski, M. (1982). Influence of interface depth upon strong ground motion, *Bull. Seism. Soc. Am.* **72,** 597–614.

England, R., F. J. Sabina, and I. Herrera (1980). Scattering of *SH* waves by surface cavities of arbitrary shape using boundary methods, *Phys. Earth. Planet. Interiors* **21,** 148–157.

Esquivel, J. A. and F. J. Sánchez-Sesma (1980). Effects of canyon topography on dynamic soil-bridge interaction for incident plane *SH* waves, *Proc. World Conf. Earthquake Engrg. 7th,* Istanbul, Turkey, **2,** 153–160.

Esteva, L. (1977). Microzoning: models and reality, *Proc. World Conf. Earthquake Engrg, 6th,* New Delhi, India.

Harmsen, S. C. and S. T. Harding (1981). Surface motion over a sedimentary valley for incident plane *P* and *SV* waves, *Bull. Seism. Soc. Am.* **71,** 655–670.

Herrera, I. (1964). A perturbation method for elastic wave propagation: 1-nonparallel boundaries, *J. Geophys. Res.* **69,** 3845–3851.

Herrera, I. (1984). *Boundary Methods: An Algebraic Theory*, Pitman Advanced Publishing Program, Boston, Massachusetts.

Hudson, J. A. (1967). Scattered surface waves from a surface obstacle, *Geophys. J. R. Astr. Soc.* **13,** 441–458.

Hudson, D. E. (1972). Local distribution of strong earthquake motion, *Bull. Seism. Soc. Am.* **62,** 1765–1786.

Hudson, J. A. and D. M. Boore (1980). Comments on "scattered surface waves from a surface obstacle," *Geophys. J. R. Astr. Soc.* **60,** 123–127.

Kupradze, V. D. (1965). *Potential Methods in the Theory of Elasticity*, Israel Program for Scientific Translations, Jerusalem, Israel.

Lamb, H. (1904). On the propagation of tremors over the surface of an elastic solid, *Phil. Trans. Royal Soc. (Lond.)* **A203,** 1–42.

Lee, V. W. (1982). A note on the scattering of elastic plane waves by a hemispherical canyon, *Soil Dynamics and Earthquake Engineering* **1,** 122–129.

McIvor, I. K. (1969). Two-dimensional scattering of a plane compressional wave by surface imperfections, *Bull. Seism. Soc. Am.* **59,** 1349–1364.

Mow, C. A. and Y. H. Pao (1971). The diffraction of elastic waves and dynamic stress concentrations, *Report R-482-PR,* The Rand Corporation, Santa Mónica, California.

Poceski, A. (1969). The ground effects on the Skopje, July 26, 1963 earthquake, *Bull. Seism. Soc. Am.* **59,** 1–29.

Sabina, F. J. and J. R. Willis (1977). Scattering of Rayleigh waves by a ridge, *J. Geophys.* **43,** 401–419.

Sabina, F. J., I. Herrera, and R. England (1978). Theory of connectivity: application to scattering of seismic waves, *Proc. Intl. Conf. on Microzonation, 2nd,* San Francisco, California, **2,** 813–824.

Sánchez-Sesma, F. J. (1978). Ground motion amplification due to canyons of arbitrary shape, *Proc. Int. Conf.* on *Microzonation,* 2nd, San Francisco, California **2,** 729–738.

Sánchez-Sesma, F. J. (1983). Diffraction of elastic waves by three-dimensional surface irregularities, *Bull. Seism. Soc. Am.* **73,** 1621–1636.

Sánchez-Sesma, F. J. and E. Rosenblueth (1979). Ground motion at canyons of arbitrary shape under incident *SH* waves, *Int. J. Earthquake Engrg. Struct. Dyn.* **7,** 441–450.

Sánchez-Sesma, F. J., I. Herrera, and J. Avilés (1982a). A boundary method for elastic wave diffraction. Application to scattering of *SH* waves by surface irregularities, *Bull. Seism. Soc. Am.* **72,** 473–490.

Sánchez-Sesma, F. J., I. Herrera, and M. A. Bravo (1982b). Difracción de ondas *P*, *SV* y de Rayleigh en un semiespacio elástico, Instituto de Ingeniería, UNAM, México.

Sánchez-Sesma, F. J., S. Chávez-Pérez, and J. Avilés (1984). Scattering of elastic waves by three-dimensional topographies. *Proc. World. Conf. Earthquake Engrg., 8th,* San Francisco, California.

Smith, W. D. (1975). The application of finite element analysis to body wave propagation problems, *Geophys. J.* **44,** 747–768.

Sommerfeld, A. (1949). *Partial Differential Equations in Physics*, Academic Press, Inc., New York.

Trifunac, M. D. (1971). Surface motion of a semi-cylindrical alluvial valley for incident plane *SH* waves, *Bull. Seism. Soc. Am.* **61,** 1755–1770.

Wong, H. L. (1982). Effect of surface topography on the diffraction of *P*, *SV*, and Rayleigh waves, *Bull. Seism. Soc. Am.* **72,** 1167–1183.

Wong, H. L. and P. C. Jennings (1975). Effect of canyon topography on strong ground motion, *Bull. Seism. Soc. Am.* **65,** 1239–1257.

Wong, H. L. and M. D. Trifunac (1974). Scattering of plane *SH* waves by a semi-elliptical canyon, *Int. J. Earthquake Engrg. Struct. Dyn.* **3,** 157–169.

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