Computational Modelling of Free and Moving Boundary Problems

Vol. 1 Fluid Flow

Editors: L.C. Wrobel
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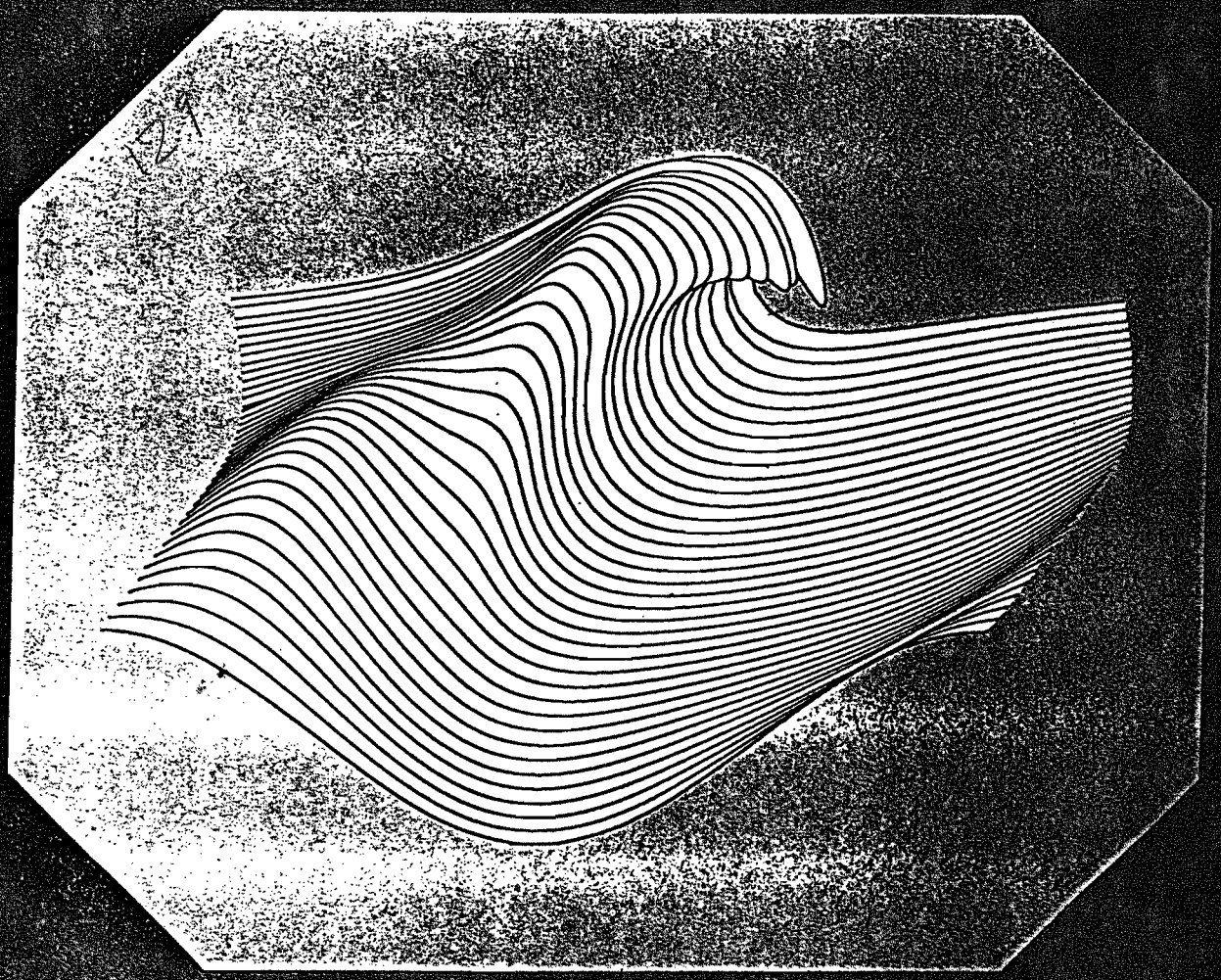

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Shock Modeling in Variable Bubble Point Problems of Petroleum Engineering

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ABSTRACT

The treatment of discontinuities is incorporated in the modeling of a gas front which invades undersaturated liquid oil, allowing in this manner a more complete treatment of these kind of problems. In this connection, a methodology of wider applicability is introduced with the following features: Systematic formulation of jump conditions; Finite differences schemes for functions with jumps and Eulerian-Langrangian approach to front tracking. Using this approach quite satisfactory numerical results are obtained.

INTRODUCTON

Several methods have been presented in the literature for representing saturation fronts accurately. A class of such methods is based on the method of characteristics and is usually limited to miscible flow problems (Douglas[1], Douglas and Russell[2], Ewing et al.[3,4], Russell[5], Herrera and Ewing[6]). Front tracking methods (Glimm et al.[7]) have some attractive features, but in its present form they are restricted to cases in which capillary effects can be ignored (Bratvedt et al.[8]). Buckley-Leverett frontal advance theory together with a linear approximation of the fractional flow function has been applied in simple reservoir studies (Kale[9]).

For the treatment of sharp discontinuities, the equations are usually separated into pressure and saturations equations and the diffusion terms associated with capillary pressure are ignored in the saturation. Then, Buckley-Leverett Theory is applied (Buckley-Leverett[10], Kale[9]). Another approach due to Glimm[7] (Bratvedt et al.[8]), is to apply the theory of non-linear conservation laws to the resulting saturation equation.

In this paper, the modeling of discontinuities which occur when a gas front invades a region of undersaturated liquid oil is discussed and in this connection, an alternative approach to sharp fronts is presented. The general formulation is equally applicable to other sharp discontinuities ("shocks"), such as oil displaced by a water front. The main contributions are:

a).- A very systematic formulation of jump conditions, including the velocity of advancing fronts;

b).- Introduction of finite difference schemes for functions with jumps and using them, development of an Eulerian-Lagragian approach to front tracking; and

c).- A rigorous mathematical setting is given for more general problems.

The case of an advancing front of gas into undersaturated liquid oil has been treated previously (Raghavan[11] and Camacho and Raghavan[12,13]), but to our knowledge this is the first time that jumps are incorporated in the model. The modeling procedure followed up to now, which does not incorporate jumps, in what follows will be called the "traditional approach".

JUMP CONDITIONS

To give to our developments a sound and firm mathematical basis, we start from first principles. The basis of the fundamental equations that govern the flow and transport of fluids in a reservoir are the balance equations of Continuous Mechanics. The synthesis of this theory that has been given by Allen, Herrera, and Pinder[14]; and by Herrera and Allen[15], are very convenient for our purposes. In the case of multi-phase systems, each phase α

moves with its own velocity $\underline{v}^{\alpha}$. The balance laws satisfied by any intensive property ψ^{α} belonging to such phase, are:

$$\psi^{\alpha}_{t} + \nabla\cdot(\psi^{\alpha}\underline{v}^{\alpha}) - \nabla\cdot\underline{\tau}^{\alpha} = g^{\alpha} \qquad (1a)$$

and

$$[\psi^{\alpha}(\underline{v}^{\alpha}-\underline{v}_{\Sigma}) - \underline{\tau}^{\alpha}]\cdot\underline{n} = g^{\alpha}_{\Sigma} \qquad (1b)$$

Equations (1a) and (1b), are the general "differential balance law" and the general "jump condition", respectively. The vector $\underline{\tau}^{\alpha}$, is the flux of ψ^{α} across surfaces in space, while the quantities g^{α} and g^{α}_{Σ} represent external supply of ψ^{α} (Allen, Herrera and Pinder[14]; Herrera and Allen[15]), per unit volume and unit time, in the case of g^{α}, while g^{α}_{Σ} represents external supply of ψ^{α} through the discontinuity, per unit area and unit time.

The black oil (or beta) model that will be considered is based on the following hypothesis:

a).- There are three phases: water, liquid oil and gas;
b).- Water and oil are immiscible, while gas is soluble only in liquid oil; i.e. the water and gas phases consist of only one component, while the liquid oil is made of two components: dissolved gas and non-volatile oil.
c).- No physical diffusion is present (this includes both, molecular diffusion and that induced by the randomness of the porous media).

With each one of the components there is associated one intensive property, which represents the mass per unit volume of that component. Thus, one can write:

$$\psi^{w}=\phi S_{w}\rho_{w},\ \bar{\psi}^{o}=\phi S_{o}\bar{\rho}_{o},\ \bar{\psi}^{dg}=\phi S_{o}\bar{\rho}_{dg},\ \psi^{g}=\phi S_{g}\rho_{g} \qquad (2)$$

for the intensive properties associated with water, non-volatile oil, dissolved gas and gas in the gas phase, respectively. Also, the flux $\underline{\tau}$ corresponding to each one of these intensive properties vanishes identically, since no physical diffusion is present. Applying Equation (1a) to each one of these intensive properties on gets

$$(\phi S_{w}\rho_{w})_{t} + \nabla\cdot(\phi \rho_{w} S_{w}\underline{v}^{w}) = 0 \qquad (3)$$

$$(\phi S_{o}\bar{\rho}_{o})_{t} + \nabla\cdot(\phi \bar{\rho}_{o} S_{o}\underline{v}^{o}) = 0 \qquad (4)$$

$$(\phi\, S_o \bar{\rho}_{dg})_t + \nabla\cdot(\phi\, \bar{\rho}_{dg} S_o \underline{v}^o) = g^o_{Ig} \qquad (5a)$$

$$(\phi\, S_g \rho_g)_t + \nabla\cdot(\phi\, \rho_g S_g \underline{v}^g) = g^g_{Io} \qquad (5b)$$

where no extraction terms have been included, so that for each one of the components the external supply terms vanish, except for those corresponding to the interchange of mass between gas and dissolved gas. Thus, g^o_{Ig} is the mass of gas that is dissolved in the liquid oil per unit volume per unit time, while g^g_{Io} is the mass of dissolved oil that goes into the gas phase per unit volume per unit time. Clearly

$$g^o_{Ig} + g^g_{Io} = 0 \qquad (6)$$

Adding up Equations (5), one gets

$$\{\phi\, (S_o \bar{\rho}_{dg} + S_g \rho_g)\}_t + \nabla\cdot\{\phi(\bar{\rho}_{dg} S_o \underline{v}^o + \rho_g S_g \underline{v}^g)\} = 0 \qquad (7)$$

Introducing the formation volume factors, Darcy's Law, as well as the relation

$$\rho = \frac{\rho_{gSTC}}{\rho_{oSTC}} R_s \bar{\rho}_o \qquad (8)$$

the system of Equations (3), (4) and (7), becomes the familiar system of equations of black oil:

$$\nabla\cdot[\lambda_w(\nabla p_w - \gamma_w \nabla z)] = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) \qquad (9a)$$

$$\nabla\cdot[\lambda_o(\nabla p_o - \gamma_o \nabla z)] = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) \qquad (9b)$$

$$\nabla\cdot[R_s \lambda_o(\nabla p_o - \gamma_o \nabla z) + \lambda_g(\nabla p_g - \gamma_g \nabla z)]$$

$$= \frac{\partial}{\partial t}\left[\phi\left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right] \qquad (9c)$$

In a similar fashion applying Equations (1b), to each one of the four components, it is obtained:

$$[\phi\, \rho_w S_w(\underline{v}^w - \underline{v}_\Sigma)]\cdot\underline{n} = 0 \qquad (10a)$$

$$[\phi\, \bar{\rho}_o S_o(\underline{v}^o - \underline{v}_\Sigma)]\cdot\underline{n} = 0 \qquad (10b)$$

$$[\phi\, \bar{\rho}_{dg} S_o(\underline{v}^o - \underline{v}_\Sigma)]\cdot\underline{n} = g^o_{\Sigma g} \qquad (11a)$$

$$[\phi\, \rho_g S_g(\underline{v}^g - \underline{v}_\Sigma)]\cdot\underline{n} = g^g_{\Sigma o} \qquad (11b)$$

In addition, Darcy's Law requires:

$$[p_l] = 0 \; ; \; l = w, o, g \qquad (12)$$

Above, the quantities $g^{o}_{\Sigma g}$ and $g^{g}_{\Sigma o}$ stand for the exchange of mass between the gaseous phase and the component of dissolved gas which is contained in the liquid oil phase. Mass conservation requires that:

$$g^{o}_{\Sigma g} + g^{g}_{\Sigma o} = 0 \qquad (13)$$

So that, adding up Equations (13), it is obtained:

$$[\phi\, \bar{\rho}_{dg} S_o \underline{v}^{o} + \phi\, \rho_g S_g \underline{v}^{g}]\cdot \underline{n} - [\phi\, \bar{\rho}_{dg} S_o + \phi\, \rho_g S_g]\underline{v}_{\Sigma}\cdot \underline{n} = 0 \qquad (14)$$

Since the pressures of the different phases are continuous, so is the porosity ϕ and ρ_w. This allows cancelling these factors in Equations (10) and (14). Thus, introducing the volume factors, they can be written as:

$$[S_w]\underline{v}_{\Sigma}\cdot \underline{n} - [S_w \underline{v}^{w}]\cdot \underline{n} = 0 \qquad (15a)$$

$$[S_o/B_o]\underline{v}_{\Sigma}\cdot \underline{n} - [(S_o/B_o)\underline{v}^{o}]\cdot \underline{n} = 0 \qquad (15b)$$

$$[R_s S_o/B_o + S_g/B_g]\underline{v}_{\Sigma}\cdot \underline{n} - [(R_s S_o/B_o)\underline{v}^{o} + (S_g/B_g)\underline{v}^{g}]\cdot \underline{n} = 0 \qquad (15c)$$

Equations (15), together with (12), constitute the desired system of jump conditions for the three-phase (four component) oil reservoir. They relate the jumps of the physical variables with the velocity $\underline{v}_{\Sigma}$ of the advancing front. Observe that, in general, the jumps of the volume factor B_o and of R_s may be non-zero since they are functions of bubble point, in addition to pressure.

THE VELOCITY OF THE ADVANCING FRONT

Thus far the discussion has been completely general. Consider now an advancing front of gas into a non-saturated liquid oil. For this case, taking the unit normal $\underline{n}$ to Σ in the opposite sense to $\underline{v}^{g}$ (i.e. $\underline{v}_g\cdot\underline{n} < 0$), it is seen that

$$[S_g/B_g] = S_g/B_g \qquad \text{and} \qquad [(S_g/B_g)\underline{v}^{g}]\cdot \underline{n} = (S_g/B_g)\underline{v}^{g}\cdot \underline{n} \qquad (16a)$$

and

$$[(R_s S_o/B_o)\underline{v}^{o}]\cdot \underline{n} = \dot{R}_s[(S_o/B_o)\underline{v}^{o}]\cdot \underline{n} + [R_s]\dot{\overline{(S_o/B_o)\underline{v}^{o}}}\cdot \underline{n} \qquad (16b)$$

Writing

$$\underline{v}_{\Sigma}\cdot \underline{n} = \eta\, \underline{v}_g\cdot \underline{n} \qquad \underline{v}_o\cdot \underline{n} = \zeta\, \underline{v}_g\cdot \underline{n} \qquad \text{and} \qquad \omega = \frac{S_o B_g}{B_o S_g} \qquad (17)$$

Equations (15b) and (15c) can be written as :

$$\eta = 1 - [R_s]\frac{[\zeta]}{[\omega]}\omega_+\omega_- ; \qquad \dot{\zeta} = \eta - \dot{\omega}\frac{[\zeta]}{[\omega]} \qquad (18)$$

Observe that, in view of Equation (18), η represents the factor by which the advancing gas front is slowed down with respect to the particle velocity of the gas phase. It is similar to the factor F_w' which occurs in the Buckley-Leverett frontal advance theory (Kale[9]). In general, $\eta > \dot{\zeta}$, since otherwise the gas front is not really advancing with respect to the unsaturated oil. Therefore, $\frac{[\zeta]}{[\omega]} > 0$, by virtue of the second of Equations (18).

NUMERICAL FORMULATION OF JUMP CONDITIONS

Equations (18) are informative and permit acquiring insight on the manner in which the advance of the gas front occurs. However, the jump conditions, in the manner in which they are presented in Equations (18), are not in their most convenient form to be used in numerical applications. In this Section they will be transformed into the form they will be used in the numerical formulations that follow.

Introducing the mobilities, Equations (16) can be written as:

$$[\lambda_w(\nabla p_w - \gamma_w \nabla z)]\cdot\underline{n} + \left[\frac{\phi S_w}{B_w}\right]\underline{v}_\Sigma\cdot\underline{n} = 0 \qquad (19a)$$

$$[\lambda_o(\nabla p_o - \gamma_o \nabla z)]\cdot\underline{n} + \left[\frac{\phi S_o}{B_o}\right]\underline{v}_\Sigma\cdot\underline{n} = 0 \qquad (19b)$$

$$[R_s\lambda_o(\nabla p_o - \gamma_o \nabla z) + \lambda_g(\nabla p_g - \gamma_g \nabla z)]\cdot\underline{n} + \left[\phi\left(R_s\frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right]\underline{v}_\Sigma\cdot\underline{n} = 0 \qquad (19c)$$

For simplicity, in what follows only two phases will be considered: oil and gas. In addition, only a 1-D formulation will be presented, so that gravity effects will be left out and it will be assumed that the gas moves towards the left (i.e., $v_\Sigma < 0$). In this case Equations (19) become:

$$\left[\lambda_o\frac{\partial p_o}{\partial x}\right] + \left[\frac{\phi S_o}{B_o}\right]v_\Sigma = 0 \qquad (20a)$$

$$\left[R_s\lambda_o\frac{\partial p_o}{\partial x} + \lambda_g\frac{\partial p_g}{\partial x}\right] + \left[\phi\left(R_s\frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right]v_\Sigma = 0 \qquad (20b)$$

Since $\lambda_g = 0$ on $\Sigma-$, it is clear that

$$\left[\lambda_g \frac{\partial p_g}{\partial x}\right] = \left(\lambda_g \frac{\partial p_g}{\partial x}\right)_{\Sigma+}$$

and Equation (20b) can be written as:

$$\left[R_s \lambda_o \frac{\partial p_o}{\partial x}\right] + \left[\phi\left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right] v_\Sigma = -\left(\lambda_g \frac{\partial p_g}{\partial x}\right)_{\Sigma+} \qquad (21)$$

Using the general relation $[ab]=\dot{a}[b]+[a]\dot{b}$, it is obtained:

$$\dot{\lambda}_o \left[\frac{\partial p_o}{\partial x}\right] + \left[\frac{\phi S_o}{B_o}\right] v_\Sigma = -\left[\lambda_o\right]\overline{\frac{\partial p_o}{\partial x}} \qquad (22a)$$

$$\dot{\overline{R_s \lambda_o}}\left[\frac{\partial p_o}{\partial x}\right] + \left[\phi\left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right] v_\Sigma = -\left(\lambda_g \frac{\partial p_g}{\partial x}\right)_{\Sigma+} - \left[R_s \lambda_o\right]\overline{\frac{\partial p_o}{\partial x}} \qquad (22b)$$

Introducing the notation

$$\Delta = \left[\frac{\partial p_o}{\partial x}\right]; \qquad m = \overline{\frac{\partial p_o}{\partial x}} \qquad (23a)$$

it is seen that

$$\left(\frac{\partial p_g}{\partial x}\right)_{\Sigma+} = m + \Delta/2 \qquad (23b)$$

Observe that capillary pressure has been neglected when writing Equation (23b). Then, the system of Equations (22) can be easily transformed into:

$$\dot{\lambda}_o \Delta + \left[\frac{\phi S_o}{B_o}\right] v_\Sigma + \left[\lambda_o\right] m = 0 \qquad (24a)$$

$$\left(2\dot{\overline{R_s \lambda_o}} + \lambda_{g+}\right)\Delta + 2\left[\phi\left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right] v_\Sigma + 2\left(\lambda_{g+} + [R_s \lambda_o]\right) m = 0 \qquad (24b)$$

For the numerical treatment, Equations (24) will be used as system of equations for the unknowns Δ, v_Σ and m, all of them defined on the space-time surface of discontinuity $\Sigma(t)$. However, this system is not determined since it is 2 by 3. The additional equation that

is needed, will be obtained weighting the differential equations (9) in a suitable manner, as it is explained in the next Section.

THE WEIGHTED EQUATIONS

The one-dimensional version of Equations (9b and c), is:

$$\frac{\partial}{\partial x}\left(\lambda_o \frac{\partial p_o}{\partial x}\right) - \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) = 0 \qquad (25a)$$

$$\frac{\partial}{\partial x}\left(R_s \lambda_o \frac{\partial p_o}{\partial x} + \lambda_g \frac{\partial p_g}{\partial x}\right) - \frac{\partial}{\partial t}\left\{\phi\left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g}\right)\right\} = 0 \qquad (25b)$$

In this Section these equations will weighted using convenient space-time weights, deriving in this manner equations suitable for discretization. The weights that will be chosen lead to what is essentially a cells method.

It will be assumed that $Cu = -V_\Sigma \Delta t/\Delta x < 1$. The position of the gas front will be denoted by $x_\Sigma(t)$. The space interval will be divided into a finite number of equally spaced cells. Taking time t_n as starting time, a procedure for contructing the solution at time t_{n+1} will be developed. The index "i" will be reserved to denote the cell containing the front at time t_{n+1}. Since $Cu<1$ and $V_\Sigma<0$, only two cases must be distinguished. Either, during the time interval (t_n, t_{n+1}) the gas front does not cross any inter-cell boundary (Case A); or it crosses one inter-cell boundary (Case B). Observe that in Case B the inter-cell boundary that is crossed is $x_{i+1/2}$.

Case A

The system of space-time weights to be used in cell "i" (i.e.; $[x_{i-1/2}, x_{i+1/2}]$), is:

$$w_1^o(x,t) = 1 \qquad (26A)$$

$$w_2^o(x,t) = \begin{cases} 0\,, & x < x_\Sigma(t) \\ 1\,, & x > x_\Sigma(t) \end{cases} \qquad (26B)$$

for the oil and

$$w^g(x,t) = \begin{cases} 0\,, & x < x_\Sigma(t) \\ 1\,, & x > x_\Sigma(t) \end{cases} \qquad (26C)$$

for the gas. The resulting equations are:

$$\int_{t_n}^{t_{n+1}} \left(\lambda_o \frac{\partial p_o}{\partial x} \right)_{l+1/2} dt + \int_{x_{l-1/2}}^{x_{l+1/2}} \left(\phi \frac{S_o}{B_o} \right)^n dx =$$

$$\int_{t_n}^{t_{n+1}} \left(\lambda_o \frac{\partial p_o}{\partial x} \right)_{l-1/2} dt + \int_{x_{l-1/2}}^{x_{l+1/2}} \left(\phi \frac{S_o}{B_o} \right)^{n+1} dx \qquad (27A)$$

$$\int_{t_n}^{t_{n+1}} \left(\lambda_o \frac{\partial p_o}{\partial x} \right)_{l+1/2} dt + \int_{x_{\Sigma(t_n)}}^{x_{l+1/2}} \left(\phi \frac{S_o}{B_o} \right)^n dx =$$

$$\int_{t_n}^{t_{n+1}} \left(\lambda_o \frac{\partial p_o}{\partial x} + \phi \frac{S_o}{B_o} V_\Sigma \right)_{\Sigma(t)} dt + \int_{x_{\Sigma(t_{n+1})}}^{x_{l+1/2}} \left(\phi \frac{S_o}{B_o} \right)^{n+1} dx \qquad (27B)$$

$$\int_{t_n}^{t_{n+1}} \left\{ R_s \lambda_o \frac{\partial p_o}{\partial x} + \lambda_g \frac{\partial p_g}{\partial x} \right\}_{l+1/2} dt + \int_{x_{\Sigma(t_n)}}^{x_{l+1/2}} \left\{ \phi \left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g} \right) \right\}^n dx =$$

$$\int_{t_n}^{t_{n+1}} \left\{ R_s \lambda_o \frac{\partial p_o}{\partial x} + \lambda_g \frac{\partial p_g}{\partial x} + \phi \left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g} \right) V_\Sigma \right\}_{\Sigma(t)} dt + \int_{x_{\Sigma(t)}}^{x_{l+1/2}} \left\{ \phi \left(R_s \frac{S_o}{B_o} + \frac{S_g}{B_g} \right) \right\}^{n+1} dx$$

(27c)

In these equations the usual notation for line integrals has been used. In particular $dt = (1+V_\Sigma^2)^{-1/2} d\sigma$, where $d\sigma$ is the length in space-time.

Equations (27) constitute a system of three equations for cell "l", which must be coupled with the system of jump conditions (24) and the equations of the remaining cells (two for each cell, when a two-phase system is considered), to obtain a determined system. Assume, for the sake of definiteness, that we are solving for pressure and saturation of the gas.Cells, other than cell "l", can be treated in a standard manner, solving for example, for oil pressure and gas saturation at the center of the cell. However,

cell "i" must be treated in a special manner. We have available the system of five equations constituted by Equations (24) and (27). A possible choice (and this one was used in the present study) of the corresponding five unknowns is: the jump and average of the pressure gradient (Δ and m, respectively), the velocity v_Σ of the gas front (these three unknown functions defined on $\Sigma(t)$), the saturation and the pressure. Then, one can solve the value p_{oi}^{n+1} at the center of cell "i" and the value $S_{g\Sigma}^{n+1}$ of gas saturation at the gas front, together with Δ^{n+1}, m^{n+1} and v_Σ^{n+1}. A special feature of this procedure is that one does not solve for saturation at the center of cell "i".

Case B

The system of space-time weights to be used in cell "i" (i.e.; $[x_{i-1/2}, x_{i+1/2}]$), is:

$$w_1^o(x,t) \equiv 1 \qquad (28A)$$

for the oil and

$$w_1^g(x,t) = \begin{cases} 0\ ,\ x < x_\Sigma(t) \\ 1\ ,\ x > x_\Sigma(t) \end{cases} \qquad (28B)$$

for the gas. The system of space-time weights to be used in cell "i+1" (i.e.; $[x_{i+1/2}, x_{i+3/2}]$), is:

$$w_2^o(x,t) \equiv 1 \qquad (28C)$$

for the oil and

$$w_2^g(x,t) = \begin{cases} 0\ ,\ x < x_\Sigma(t) \\ 1\ ,\ x > x_\Sigma(t) \end{cases} \qquad (28D)$$

for the gas. In addition, a space-time weight with support in the union of cells "i" and "i+1", is applied to the oil equation. It is defined by

$$w_3^o(x,t) = \begin{cases} 0\ ,\ x < x_\Sigma(t) \\ 1\ ,\ x > x_\Sigma(t) \end{cases} \qquad (28E)$$

Corresponding to these five weighting functions, five equations are derived for the union of cells "i" and "i+1". Putting them together with the jump conditions (Equations 24) a system of seven equations is obtained. The corresponding seven unknowns selected for the applications of the present paper were:

the jump and average of the pressure gradient at the gas front at time t_{n+1}, the velocity of the advancing front v_{Σ}, the values p_{oi}^{n+1} and p_{oi+1}^{n+1}, of the oil pressure at cells "i" and "i+1", respectively, and the saturations of the gas S_{gi+1}^{n+1} and $S_{g\Sigma}^{n+1}$ at the center of cell "i+1" and at the gas front, respectively. Again, as in Case A, a special feature of the procedure is that one does not solve for the saturation at the center of cell "i".

NUMERICAL RESULTS

As a test for the formulation presented in this paper, the numerical results that were obtained using it in a black oil simulator, neglecting capillary pressure, are presented.

A linear reservoir is considered, producing at a constant oil rate of 800 m^3/day at the left boundary, and a no flow boundary at the right end (L = 550 meters) . This reservoir is composed of two zones, a left undersaturated oil zone and a right saturated zone, where the oil and gas phases coexist. Initially, the pressure is the same for all positions, S_g = 0.7 in the saturated zone, and the front is located at x= 275 meters.

The PVT properties for the oil phase consist of constant viscosity and two values of B_o and R_s, one at the left side of the front and another at the right side, respectively. For the gas phase the PVT properties used, are typical in the oil industry. Straight line relationships are used for the relative permeabilities.

In order to check the validity of the simulator results, we have considered two cases: firstly, the undersaturated zone was taken as incompressible, and secondly, a non-zero value was given to the rock compressibility.

The analytical solution for the first case in the undersaturated zone dictates a constant pressure gradient. Figure 1, shows a comparison of pressure profiles at different times for this case. Two kinds of profiles are presented. The solid lines correspond to the solution obtained when the proposed formulation is included, and the dashed lines correspond to the traditional formulation in which jumps are not considered. As already mentioned, the pressure gradient is

constant in the incompressible zone.

To correlate the transient (one phase) solution for a slightly compressible liquid with a multiphase flow solution, Raghavan[11] and, Camacho and Raghavan[12,13], have proposed the use of a function of pressure and saturation, called pseudopressure, which is defined as follows:

$$m(p) = \frac{k\ h\ W}{q} \int_{x=0}^{x=L} \left[\frac{k_{ro}}{\mu_o\ B_o} \frac{\partial P}{\partial x} \right] dx \qquad (29)$$

For a linear system containing a slightly compressible liquid, Nabor and Barham[16] have shown that the pressure drop behaves linearly with time in a log-log plot, with a slope of one-half during the transient period. Once the outer boundary is manifested in the response, the solution deviates from the straight line, moving up for a closed outer boundary and achieving a constant value, whena constant pressure outer boundary condition is prescribed. When, as in the traditional approach, the jumps are not considered in the formulation, the pseudopressure defined by Equation (29) will deviate below the straight line for a closed outer boundary, after the transient period has ended. To over come this limitation, Camacho and Raghavan[12,13] have suggested to include an additional integral in the right hand side of Equation (29). In this manner, variations in average properties are taken into account.

For a constant pressure (constant saturation) outer boundary, Equation (29) yields a constant value, equal to the position of this boundary, after the transient period has ended. Figure 2 shows the behavior of the pseudopressure for the compressible case. The solid line corresponds to the solution obtained when our formulation for the jumps is used, and the dots correspond to the traditional formulation. It can be observed the presence a straight line with one-half slope during the transient period, in the solution for both formulations. Observe that when the jumps are considered in the formulation, the pseudo pressure is equal to the position of the front, since this has a similar effect to a constant pressure outer

boundary. For the traditional formulation, the pseudopressure falls below because of the reason explained above.

The behavior of pressure at x=0 is shown in Figure 3. In Figure 4 it is presented a comparison of pressure profiles at different times. The results of Figures 3 and 4 correspond to those of Figure 2. The solid lines represent the proposed formulation and the dots the traditional formulation.

Figures 5 to 8 show the behavior of v_{Σ} ,m , Δ, y x_{Σ} versus time, respectively, for both compressible and incompressible inner zone cases. The solid lines represent the compressible case and the dots the incompressible one.

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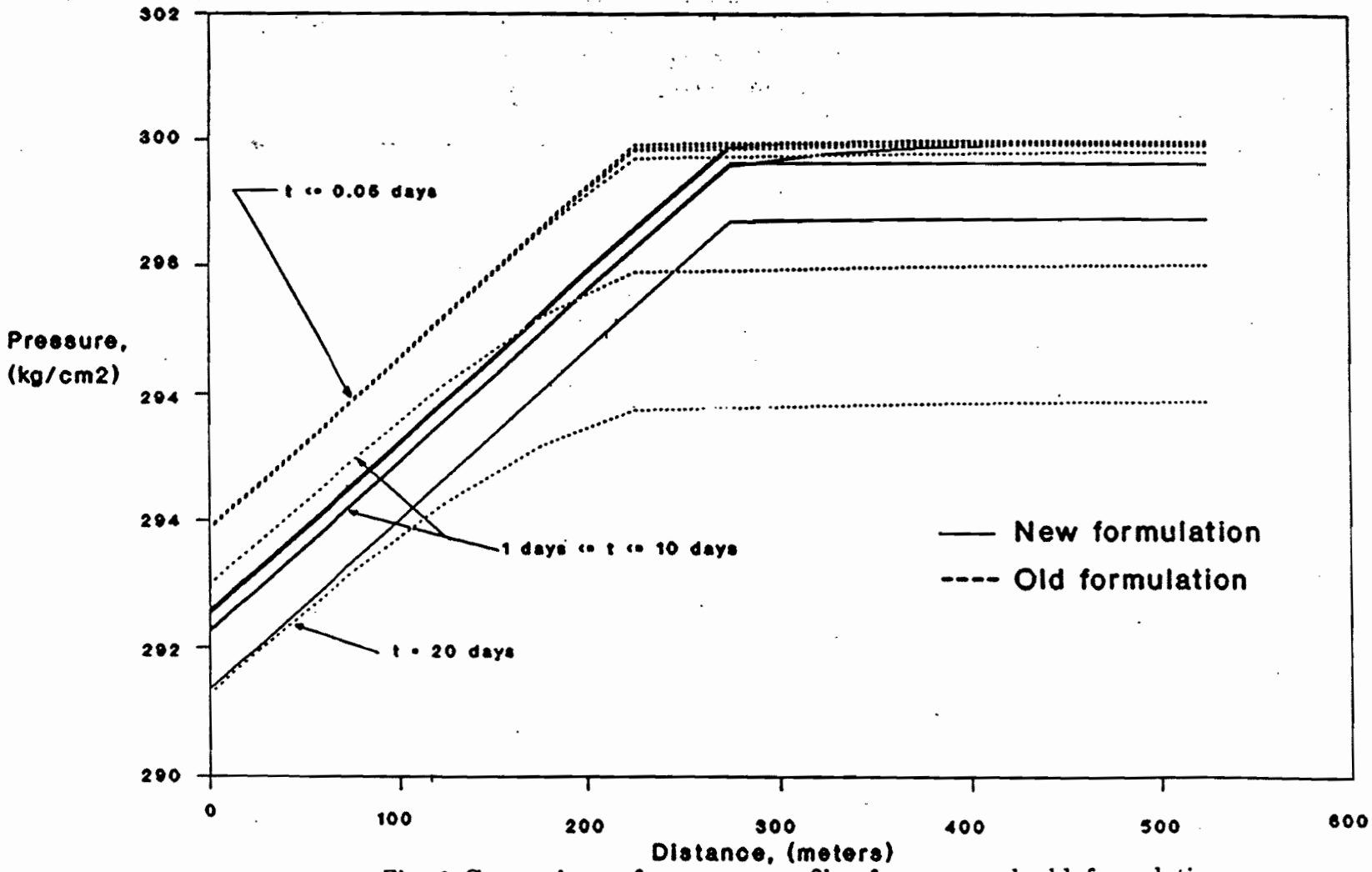

Fig. 1 Comparison of pressure profiles for new and old formulation, incompressible case.

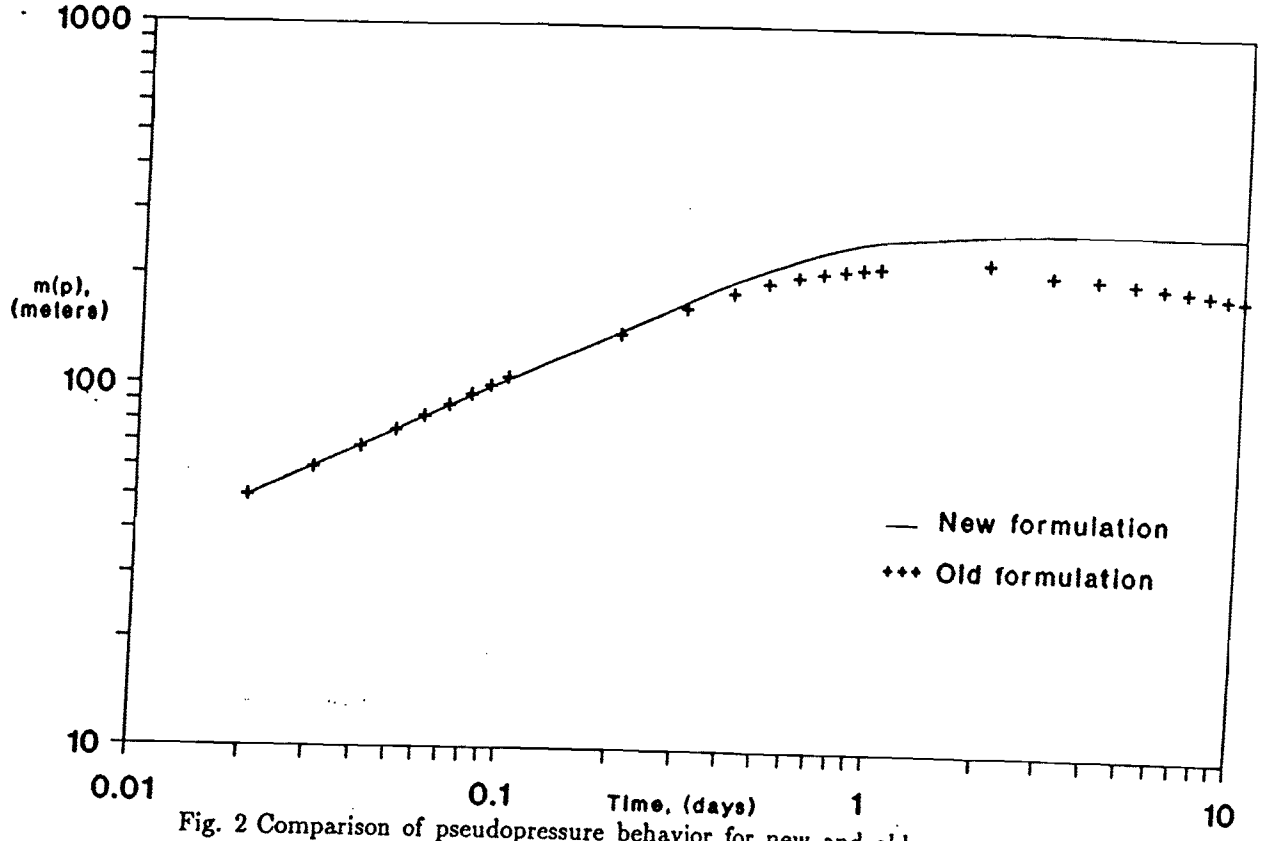

Fig. 2 Comparison of pseudopressure behavior for new and old formulations.

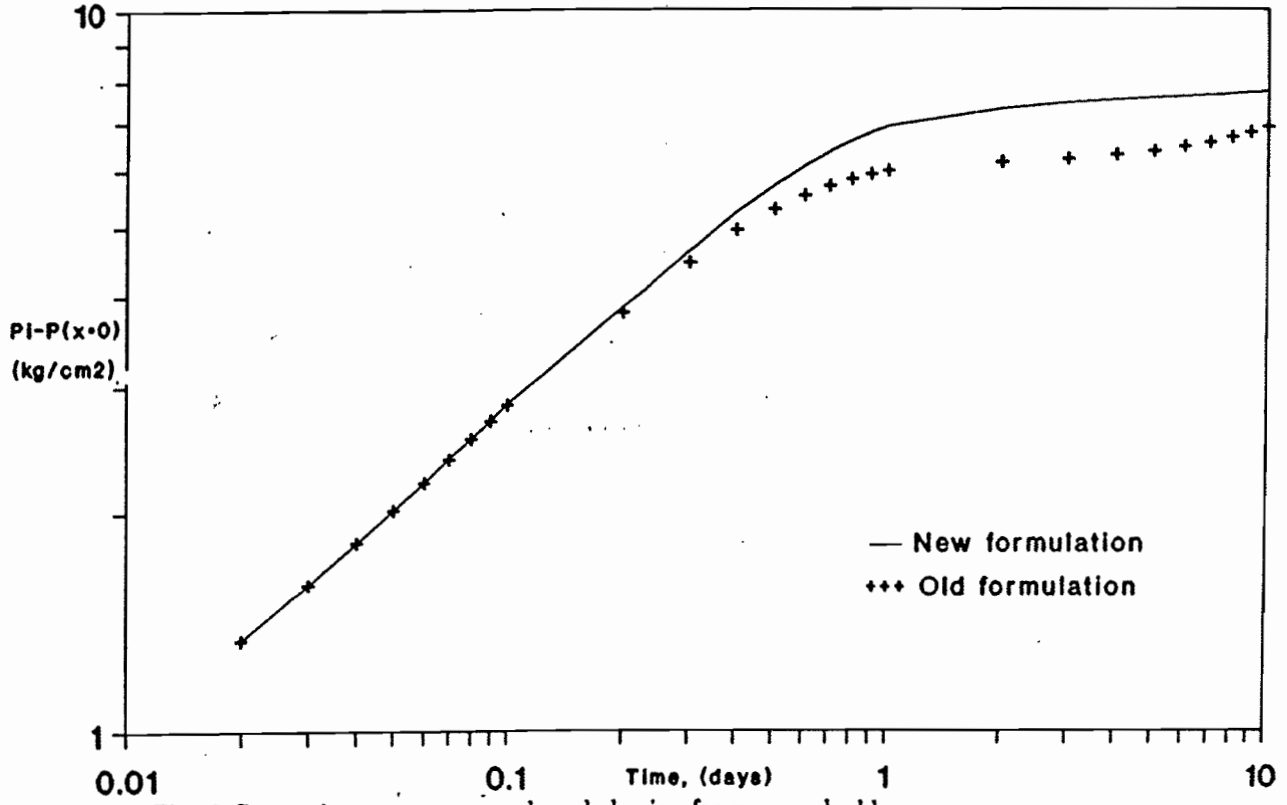

Fig. 3 Comparison on pressure drop behavior for new and old formulations.

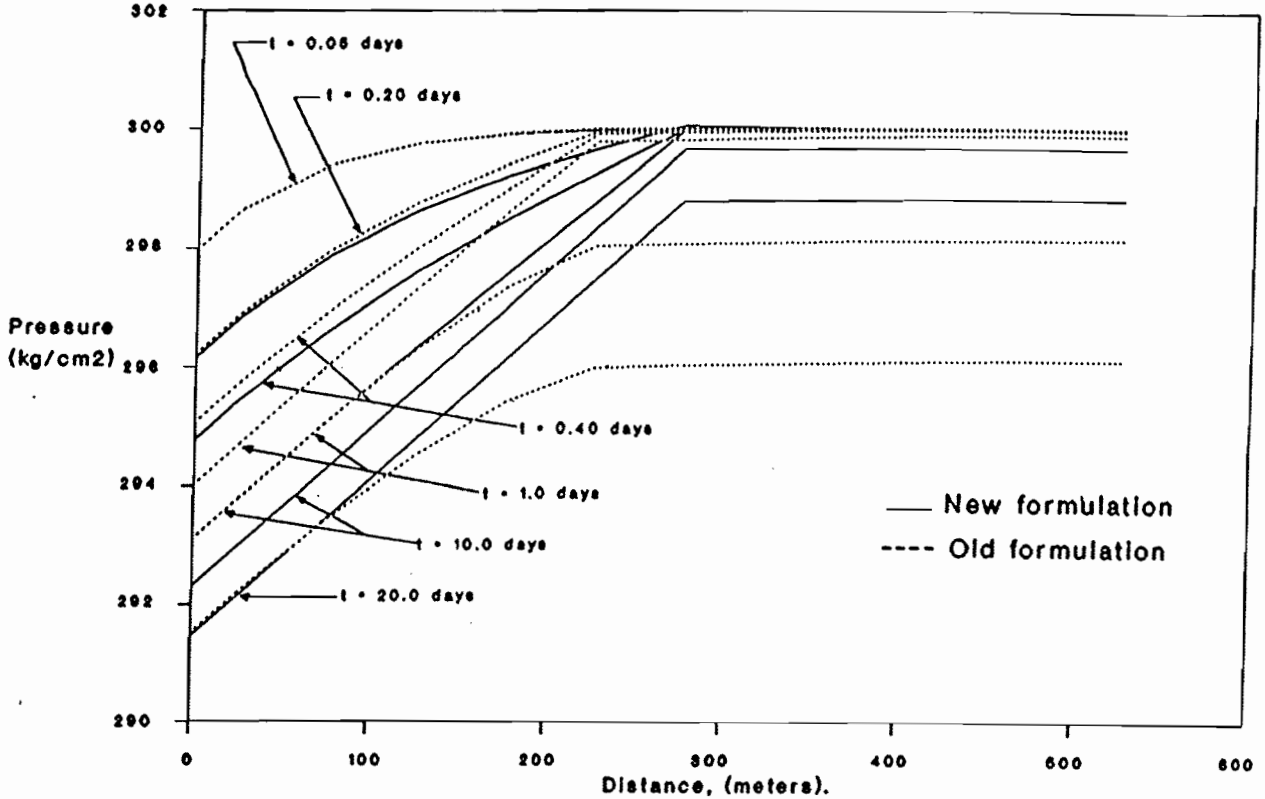

Fig. 4 Comparison of pressure profiles for new and old formulations compressible case.

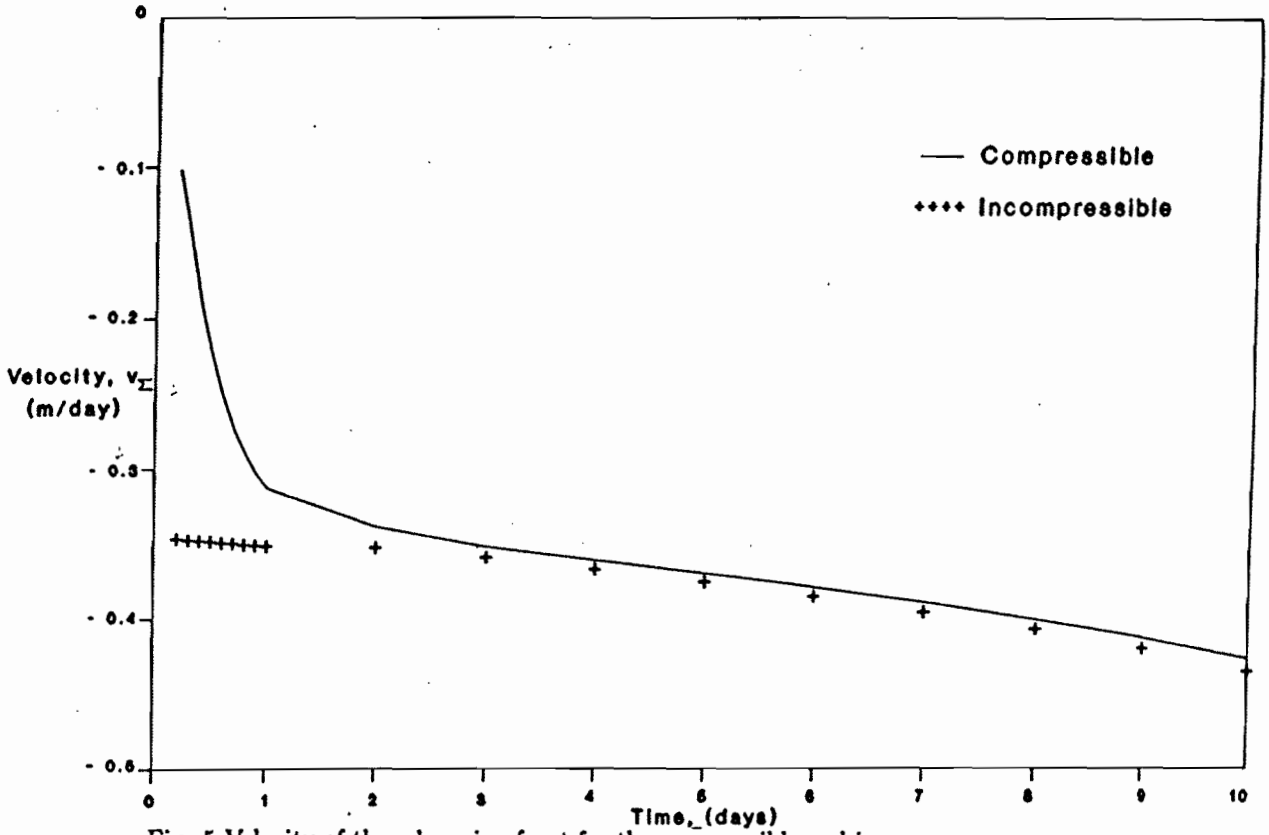

Fig. 5 Velocity of the advancing front for the compressible and incompressible cases.

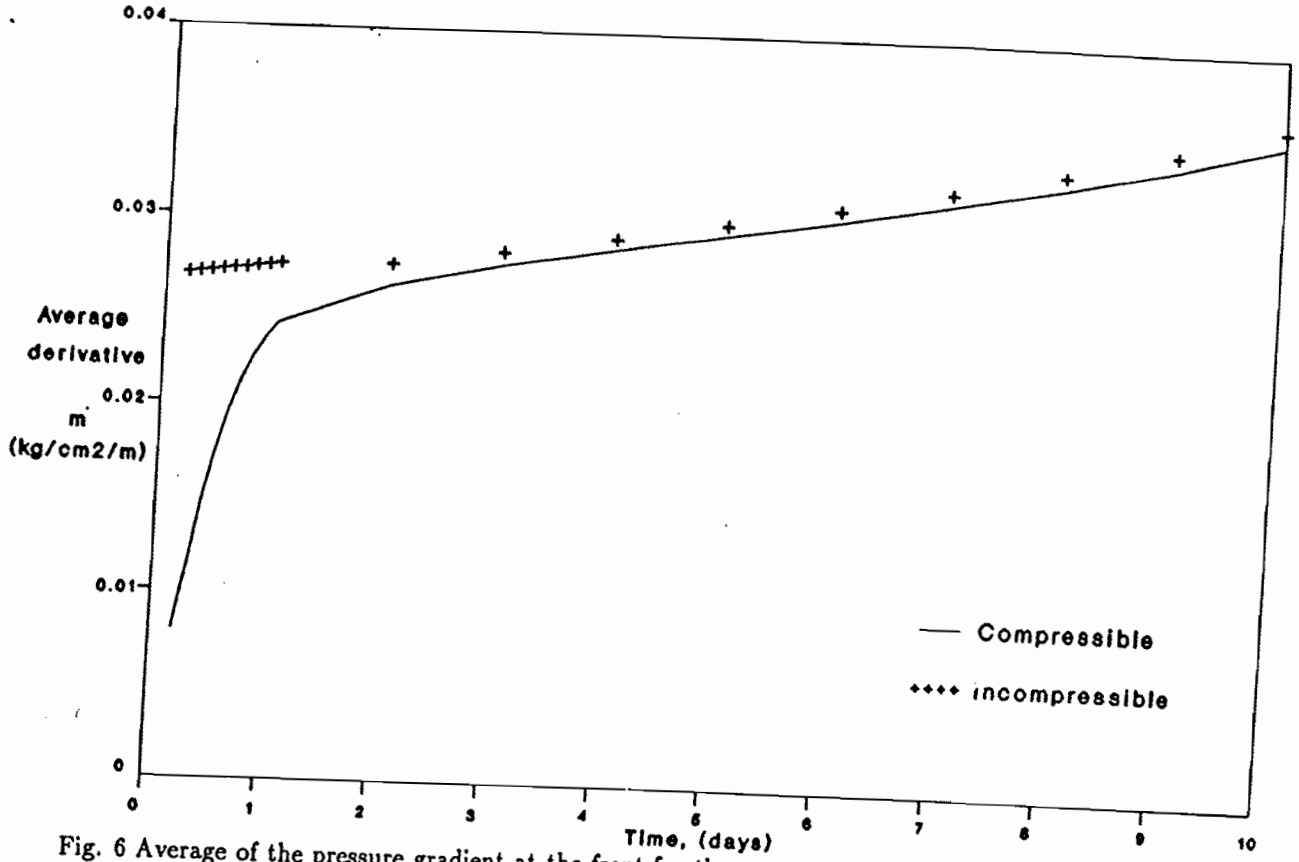

Fig. 6 Average of the pressure gradient at the front for the compressible and incompressible cases.

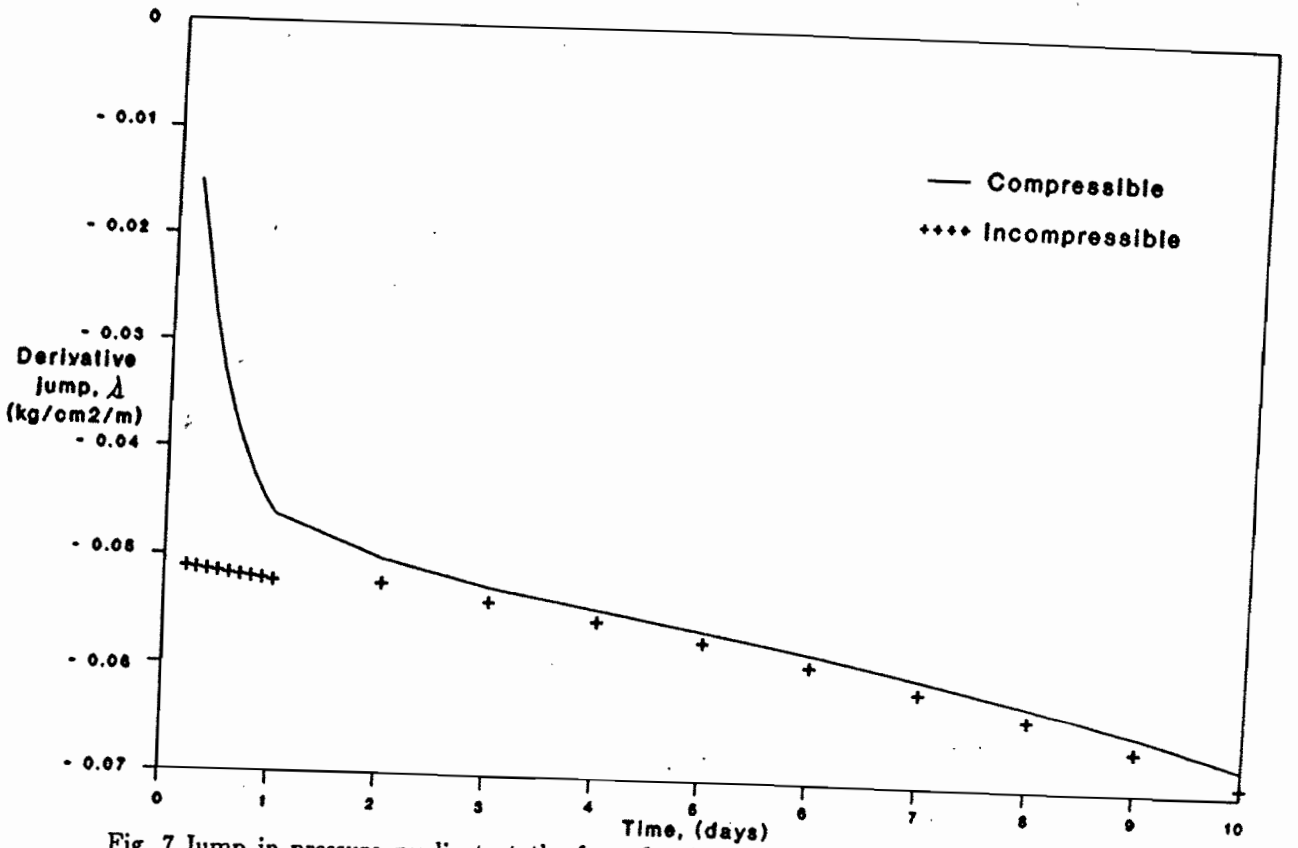

Fig. 7 Jump in pressure gradient at the front for the compressible and incompressible case.

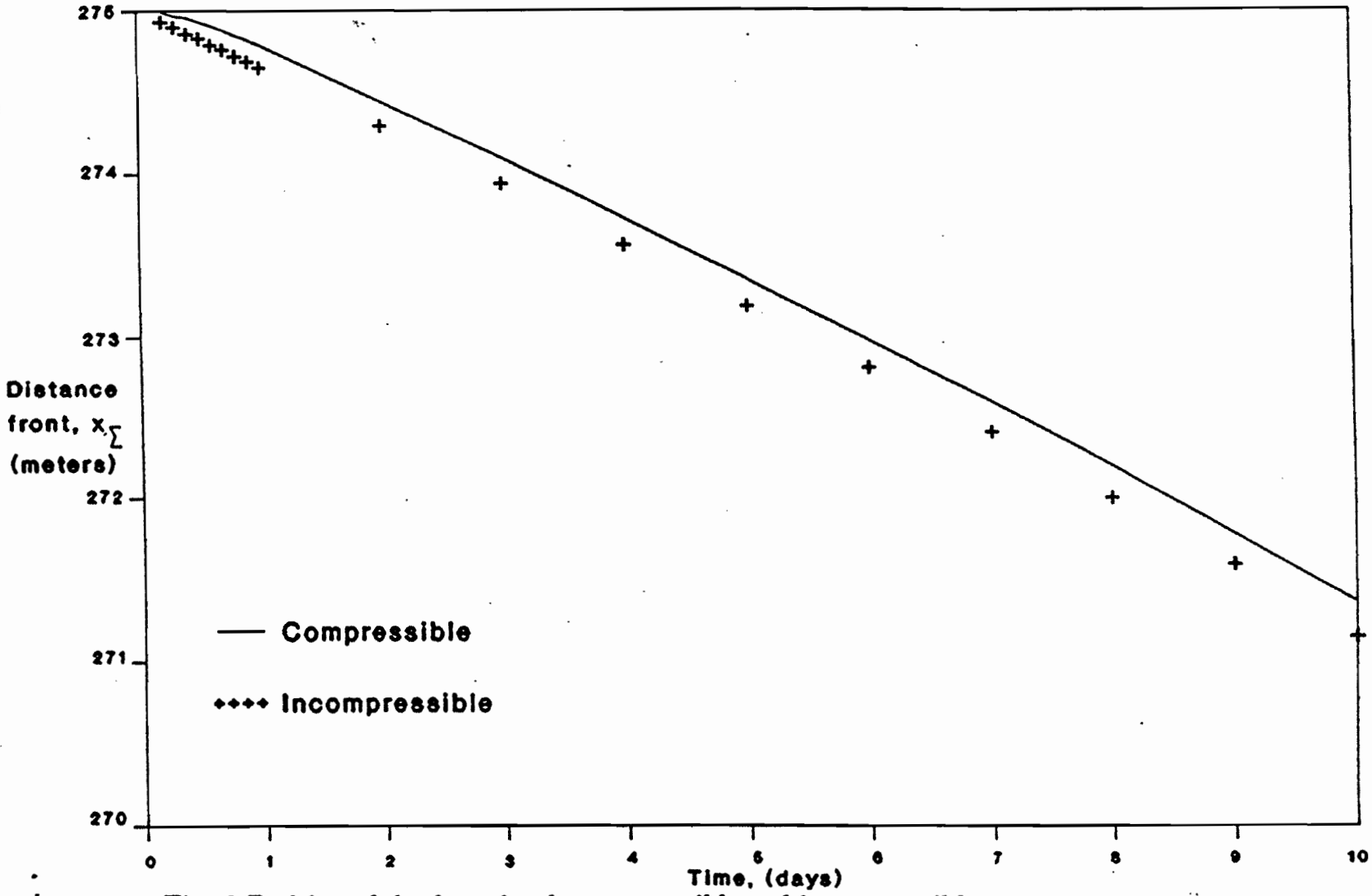

Fig. 8 Position of the front for the compressible and incompressible cases.