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Shocks in Solution Gas-Drive Reservoirs

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ABSTRACT

Black-oil models neglect diffusive mechanisms such as molecular diffusion or mechanical dispersion; this omission produces a propensity to developing shocks. This paper aims to carry out an exhaustive identification of the kind of shocks that can occur when black-oil models are applied to problems in which the bubble-point varies and to establish the conditions under which they are generated. In addition to shocks of Buckley-Leverett type, two other classes of shocks and a bifurcation mechanism, are identified. Except for shocks of Buckley-Leverett type, all other shocks may occur in the presence of capillary forces. The paper contributes to clarify several aspects of black-oil models and to understand pathologies that occur in their numerical implementation.

INTRODUCTION

The present paper stems from an effort initiated by the main author [1-3] to understanding some features of black-oil (or beta) models [4]; special attention has been given to the limitations imposed by the simplifying assumptions of such models. In particular, they do not incorporate molecular diffusion, nor mechanical dispersion. A consequence of this simplification is a property referred here as the "bubble-point conservation principle": *In the absence of a gas-phase, oil-particles conserve their dissolved gas content (oil:gas ratio, or equivalently: bubble point)*. Physically, this means that, when a gas-phase is not present, two oil particles cannot exchange dissolved gas, independently of how close they may be. This property in turn, produces a propensity of black-oil models to develop shocks which becomes apparent in problems wth variable bubble-point. In the present paper, a gas-front moving into a region occupied by undersaturated oil -as in a solution gas-drive- will be considered.

The effects we are referring to, are quite different to those analyzed by Buckley-Leverett theory and, to make our points more clear, we found convenient to place them in the general perspective of shocks that may occur in black-oil models. Thus, the present paper describes a systematic and exhaustive-analysis of the different kinds of shocks that can occur in black-oil models, and the conditions under which they are generated, without discussing details of the numerical implementation, which have been treated in previous publications [1-3] of this sequence (see also [5-7]).

In some respects, this paper is the continuation -and to some extent the culmination- of lines of thought that were initiated in [1-3]. However, the presentation here intends to be self-contained while avoiding unnecessary repetitions. Thus Buckley-Leverett theory, extensively discussed in many other papers [8-12], is not developed in detail, although some of its results are briefly described and used. Buckley-Leverett theory [8-10] deals with an important kind of shocks which occur in black-oil model applications, and Cardwell & Sheldon [11,12] explained clearly the generation mechanisms of such shocks. From a present-day perspective, Buckley-Leverett theory can be thought of as a 'hyperbolic conservation law' [5]. The interested reader is referred to [2,7] for a recent account of such developments. Additional references on this subject are given there. The results to be presented in what follows indicate that although shocks of this kind may occur only when capillary pressure is absent, this is not the case for other kinds of shocks discussed here, since the presence of capillary forces does not preclude their occurrence.

When dealing with variable bubble-point problems, in general, in which free gas may, or may not, coexist with liquid oil, the region of definition of the problem can be divided into three subregions:

a).-A region where free gas is absent, usually, undersaturated;

b).-A region where gas is present, necessarily, saturated; and

c).-A gas-front; i.e., a boundary between an undersaturated and a saturated region.

Then, a summary of the results presented here, is as follows:

-In the saturated region, the shocks that can occur are essentially of the type described by the Buckley-Leverett theory and they exist only if capillary pressure is neglected.

-In the unsaturated region, the bubble point may have jump discontinuities and such discontinuities propagate with the velocity of the oil particles.

-At a gas-front two possible situations must be distinguished:

a).- A front that advances into an undersaturated region. At such a front, in general, both the bubble point and the saturation are discontinuous and the motion of the front is retarded with respect to the gas particles.

b).- A front that recedes from an undersaturated region. At such a front, only the saturation is discontinuous and the front moves with the velocity of the gas particles. In addition, when an advancing gas front changes its sense of motion and starts to recede, the shock <u>bifurcates</u>, giving rise to two shocks: one moving with the oil velocity and the other one with the velocity of the gas.

It is important to stress that only in one case the presence of capillary pressure precludes the occurrence of shocks: in the saturated region where free gas is present, and shocks ocurring in that region are described by Buckley-Leverett theory. The other types of shocks may occur even if capillary pressure is incorporated in the model. Finally, the jump conditions that prevail at an advancing gas front are of the same kind as those that apply in Stefan problems [13].

It seems that the results presented here may be useful on several counts. Firstly, they contribute to clarify some aspects of black-oil models. For example, as mentioned before, a consequence of omitting diffusion and dispersion is the "bubble-point conservation principle". This property, leads to the preservation of discontinuities (shocks) of the oil:gas ratio. Further more, due to the <u>bubble-point conservation principle</u>, the manners in which the gas-phase can transfer gas to the liquid-oil phase, are rather restricted. In this respect, it has interest to point out that shocks occurring at an advancing gas front, constitute an additional mechanism for transferring gas from the gas-phase to oil-particles; thus, relaxing somewhat such restrictions. Finally, numerical difficulties may occur in numerical models when pathologies such as shocks, are developed. A clear understanding of them is important to design adecuate numerical treatments and effectively overcome such difficulties -previous studies did not deal with such pathologies explicitly (see for example [14-16]).

THE BLACK-OIL MODEL

For simplicity, we consider a "black-oil" or "beta" model, consisting of two phases, liquid oil and gas (whose particle velocities are denoted by $\underline{v}^o$ and $\underline{v}^g$, respectively), based on the following assumptions:

- Gas is soluble in liquid oil; i.e., the gas phase consists of only one component, while the liquid oil is made up of two components (dissolved gas and non-volatile oil). This implies that the total number of components is three and that the latter two components move with the same velocity;
- No physical diffusion is present. This includes both molecular diffusion and that induced by the randomness of the porous medium (mechanical dispersion).

Black-oil models, in general, include the possibility of non-vanishing capillary pressure, as is done here.

We use the notations $\bar{\rho}_o$ and $\bar{\rho}_{dg}$, for the effective densities of non-volatile oil and dissolved gas, respectively, together with the relation:

$$\bar{\rho}_{dg} = \bar{R}_s \bar{\rho}_o, \quad \text{where } \bar{R}_s \equiv \frac{\rho_{gSTC}}{\rho_{oSTC}} R_s \qquad (1)$$

The factor R_S is the "solution gas:oil ratio" [4]. Application of the mass conservation conditions [1,2] yields:

$$\left(\phi S_o \bar{\rho}_o\right)_t + \nabla \cdot \left(\phi \bar{\rho}_o S_o \underline{v}^o\right) = 0 \qquad (2a)$$

$$\left(\phi S_o \bar{R}_s \bar{\rho}_o\right)_t + \nabla \cdot \left(\phi \bar{R}_s \bar{\rho}_o S_o \underline{v}^o\right) = g^o_{lg} \qquad (2b)$$

$$\left(\phi S_g \bar{\rho}_g\right)_t + \nabla \cdot \left(\phi \bar{\rho}_g S_g \underline{v}^g\right) = g^g_{lo} \qquad (2c)$$

No extraction terms, due to wells, have been included and g^o_{lg} is the mass of gas that is dissolved in the liquid oil per unit volume per unit time, while g^g_{lo} is the mass of dissolved oil that goes into the gas phase per unit volume and per unit time.

Clearly

$$g^o_{lg} + g^g_{lo} = 0 \qquad (3)$$

for mass conservation.

There are situations in which it is necessary to consider discontinuous solutions. Surfaces on which discontinuities take place are usually referred to as "shocks" and will be represented by Σ; its unit normal vector will be $\underline{n}$. Mass conservation requires the jump conditions [1,2]:

$$\left[\phi \bar{\rho}_o S_o (\underline{v}^o - \underline{v}_\Sigma)\right] \cdot \underline{n} = 0 \qquad (4a)$$

$$\left[\phi \bar{\rho}_o S_o R_s (\underline{v}^o - \underline{v}_\Sigma)\right] \cdot \underline{n} = g^o_{\Sigma g} \qquad (4b)$$

$$\left[\phi \bar{\rho}_g S_g (\underline{v}^g - \underline{v}_\Sigma)\right] \cdot \underline{n} = g^g_{\Sigma o} \qquad (4c)$$

to be satisfied at shocks. Here, square brackets stand for the jump of the function inside; i.e., value on the positive side minus value on the negative one; in the understanding that the unit normal vector points towards the positive side. The velocity of the shock is $\underline{v}_\Sigma$,, while the quantities $g^o_{\Sigma g}$ and $g^g_{\Sigma o}$ are introduced to account for the exchange of mass, on Σ; between the gas and liquid oil phases. When these quantities are different from zero (as at a gas front which advances into a region of undersaturated oil), a mass exchange concentrated on the surface Σ, between the gas and liquid phases, takes place. This is in contrast with the quantities g^o_{lg} and g^g_{lo} of Eqs. (2b) and (2c), which represent a mass exchange distributed on a volume and not concentrated on a surface.

Mass conservation requires:

$$g^o_{\Sigma g} + g^g_{\Sigma o} = 0 \qquad (5)$$

In addition, Darcy's Law implies:

$$[p_l] = ; l = 0, g \qquad (6)$$

Eqs. (2) to (6), when complemented by suitable constitutive equations such as Darcy's Law for multi-phase systems, constitute a complete system of governing equations for the black-oil model. Darcy's

Law for multi-phase systems is frequently expressed in terms of Darcy velocities defined by:

$$\underline{u}_\alpha = \phi S_\alpha \underline{v}^\alpha ; \qquad \alpha = \text{o and g} \tag{7}$$

Then,

$$\rho_\alpha \underline{u}_\alpha = -\lambda_\alpha \nabla p_\alpha, \alpha = o, g \tag{8}$$

where the parameters λ_o and λ_g, are defined by

$$\lambda_\alpha = \phi\rho_\alpha \frac{k\underline{\underline{k}}_{r\alpha}}{\mu_\alpha}, \ \alpha = 0, g \tag{9}$$

Here μ_α stands for the viscosity of the different phases Note that in the presence of Eq. (2a) one can write

$$\phi S_o \overline{\rho}_o \left\{ \left(\overline{R}_s\right)_t + \underline{v}^o \cdot \nabla \overline{R}_s \right\} = g^o_{lg} \tag{10}$$

$\phi\rho_g S_g$ instead of Eq. (2b). Also, adding Eqs. (2c) and (10), one gets:

$$\phi S_o \overline{\rho}_o \left\{ \left(\overline{R}_s\right)_t + \underline{v}^o \cdot \nabla \overline{R}_s \right\} + \left(\phi S_g \rho_g\right)_t + \nabla \cdot \left(\phi \rho S_g \underline{v}^g\right) = 0 \tag{11}$$

In a similar fashion, in the presence of (4a), Eq. (4b) can bereplaced by

$$\left[\overline{R}_s\right] \overline{\phi \overline{\rho}_o S_o (\underline{v}^o - \underline{v}_\Sigma)} \cdot \underline{n} = g^o_{\Sigma g} \tag{12}$$

where

$$\overline{\overline{\rho}_o S_o (\underline{v}^o - \underline{v}_\Sigma)} \cdot \underline{n} = \tfrac{1}{2} \left[\left(\rho_o S_o \left(\underline{v}^o - \underline{v}_\Sigma\right)\right)_+ + \left(\rho_o S_o \left(\underline{v}^o - \underline{v}_\Sigma\right)\right)_- \right]$$

by virtue of Eq. (4a). Adding Eqs. (4c) and (12), one gets

$$\left[\overline{R}_s\right] \phi \overline{\overline{\rho}_o S_o (\underline{v}^o - \underline{v}_\Sigma)} \cdot \underline{n} + \left[\phi \rho_g S_g \left(\underline{v}^g - \underline{v}_\Sigma\right)\right] \underline{n} = 0 \tag{14}$$

Thus, Eqs. (2b) and (2c), can be replaced by (10) and (11), and similarly, Eqs. (4b) and (4c), can be replaced by Eqs. (12) and (14). Common expressions for the governing equations of the black-oil model could be derived from the above ones, by introducing Darcy velocities. However, we will not do that because such equations will not be used in what follows; our results are more easily derived applying the equations given here.

Now let us restrict our attention to situations in which a gas front divides the region of study into two subregions: one in which free gas is present (the "gas region") and the other in which there is only undersaturated oil (the "unsaturated oil region"). To be specific, at the gas front, the unit normal vector $(\underline{n})$ will be taken as pointing towards the gas region.

By a complete system of governing equations we mean that when such a system is complemented with appropriate initial and boundary conditions well posed problems are defined. In Section 5, initial-value problems in one dimension will be considered. The region of definition of such problems will be the interval [0,l], for t>0, and the gas region will be located at the right-hand side of the gas front. In that case, several combinations of initial and boundary conditions lead to well posed problems. For example, one such set is:

<u>a) Initial conditions.</u>

a1) The values of $\phi\rho_g S_g$ and P_o, at t=0;

a2) The bubble-point (i.e., R_s), in the region where no free gas is present.

<u>b) Boundary conditions.</u>

b1) The value of "P_o" at x=0, for $t \geq 0$;

b2) At x=0, the value of R_s at times $t \geq 0$, when $\underline{u}_o > 0$;

b3) The values of $\underline{u}_g$ *and* $\underline{s}_g$ at x = 1.

Obviously the same is true if $\underline{u}_g$ and S_g are replaced by u_o *and* S_o.

TYPES OF SHOCKS AND THEIR VELOCITIES

Buckley and Leverett treated one class of shocks, occurring in black-oil models, in their classical theory. In this section an exhaustive analysis of the different kinds of shocks that can occur in black-oil models is carried out.

To be systematic it is necessary to consider the following cases:

A). The shock occurs at a gas front, so that the gas-phase is present at one side of the shock only;

B). The shock occurs at the unsaturated oil region, so that the gas-phase is absent from both sides of the shock;

C). The shock occurs at the gas region, so that the gas-phase is present at both sides of the shock.

The shock velocities. Let us present a unified formula for the velocity of propagation of the shock, applicable to cases A) and C), in which gas is present on at least one side of the shock. To this end, define the parameters ε, δ and γ by mean of the equations

$$\left(\underline{v}_\Sigma - \underline{v}^o_+\right)\cdot \underline{n} = \varepsilon\left(\underline{v}^g_+ - \underline{v}^o_+\right)\cdot \underline{n},$$
$$\left[\underline{v}^g\right]\cdot \underline{n} = \delta\left(\underline{v}^g_+ - \underline{v}^o_+\right)\cdot \underline{n},$$
$$\left[\underline{v}^o\right]\cdot \underline{n} = -\gamma\left(\underline{v}^g_+ - \underline{v}^o_+\right)\cdot \underline{n}$$

The first of these equations expresses the relative velocity of the shock with respect to the velocity of the oil, for the positive side, as a fraction of the corresponding relative velocity of the gas.

Observe that ε, when ε<1, can be interpreted as a retardation factor. Also, in case A), strictly speaking, $\underline{v}^g_-$ is not defined, since no gas is present at the unsaturated region. However, to give a meaning to the above formulas, we define $\underline{v}^g_-$ to be zero in case A). With this convention, the following result holds.

UNIFIED FORMULA FOR THE SHOCK VELOCITIES

Assuming the porosity to be continuous, the jump conditions given by Eqs. (4b) and (4c), can be replaced by

$$\varepsilon = \frac{1+\delta\dfrac{\rho_{g-}S_{g-}}{\left[\rho_g S_g\right]}}{1+\left[R_s\right]\dfrac{\bar{\rho}_{o+}S_{o+}}{\left[\rho_g S_g\right]}}, \qquad \gamma = -\frac{\left[\bar{\rho}_o S_o\right]}{\bar{\rho}_{o-}S_{o-}}\varepsilon, \qquad (16)$$

in the system of governing equations (2) to (6).

Proof.- Eq. (14) is:

$$\left[R_s\right]\bar{\rho}_{o+}S_{o+}\left(\underline{v}^o_+ - \underline{v}_\Sigma\right)\cdot \underline{n} + \rho_{g+}S_{g+}\left(\underline{v}^g_+ - \underline{v}_\Sigma\right)\cdot \underline{n}$$
$$-\rho_{g-}S_{g-}\left(\underline{v}^g_- - \underline{v}_\Sigma\right)\cdot \underline{n} = 0$$

by virtue of Eq. (13) and the definition of the jump of a function. Thus

$$\left[R_s\right]\bar{\rho}_{o+}S_{o+}\left(\underline{v}^o_+ - \underline{v}_\Sigma\right)\cdot \underline{n} + \left[\rho_g S_g\right]\left(\underline{v}^g_+ - \underline{v}_\Sigma\right)\cdot \underline{n}$$
$$+\rho_{g-}S_{g-}\left[\underline{v}^g\right]\cdot \underline{n} = 0$$

Introducing the definitions of Eq. (15), one gets

$$\varepsilon\left[R_s\right]\bar{\rho}_{o+}S_{o+} + (\varepsilon-1)\left[\rho_g S_g\right] - \delta\rho_{g-}S_{g-} = 0$$

Solving for ε, the first of formulas (16) is obtained.

To obtain the second formula, note that the jump condition (4a) can be written as

$$\bar{\rho}_{o+}S_{o+}(\underline{v}^o_+ - \underline{v}_\Sigma \quad \underline{n} = \bar{\rho}_{o-}S_{o-}(\underline{v}^o_- - \underline{v}_\Sigma)\cdot \underline{n}$$
$$= \bar{\rho}_{o-}S_{o-}\left\{\left(\underline{v}^o_+ - \underline{v}_\Sigma\right) - \left[\underline{v}^o\right]\right\}\cdot \underline{n}$$

or

$$\left[\bar{\rho}_o S_o\right](\underline{v}^o_+ - \underline{v}_\Sigma)\cdot \underline{n} + \bar{\rho}_{o-}S_{o-}\left[\underline{v}^o\right]\cdot \underline{n} = 0$$

which becomes

$$\varepsilon\left[\bar{\rho}_o S_o\right] + \gamma\bar{\rho}_{o-}S_{o-} = 0$$

after the definitions (15) are introduced. When this equation is solved for γ, the second of Eqs. (16) follows.

Case A

Recall that $S_{g-} = 0$, necessarily. Therefore, $S_{o-} = 1$ and the Eqs. (16) become

$$\varepsilon = \frac{1}{1+\left[R_s\right]\dfrac{\bar{\rho}_{o+}S_{o+}}{\rho_{g+}S_{g+}}}, \quad \gamma = (1-S_{o+}\frac{\bar{\rho}_{o+}}{\bar{\rho}_{o-}})\varepsilon \qquad (17)$$

The first of these equations for the factor ε, was first derived in [3] (Eq. 49). It implies that $\varepsilon \le 1$, in the case we are considering, and it is necessary to distinguish two possibilities.

A gas front advancing into a region of undersaturated oil.

This situation is characterized by the fact that $\underline{v}^o_+ - \underline{v}^g_+ > 0$ and $[R_s] \rangle 0$, so that the parameter ε is a retardation factor, since it satisfies the condition $\varepsilon < 1$.

A gas front receding from a region of undersaturated oil.

This situation is characterized by the fact that

$\underline{v}^o_+ - \underline{v}^g_+ < 0$ and $[R_s] = 0$, necessarily, because as the gas front recedes it leaves saturated oil behind. Thus the oil is saturated on both sides of the gas front. Eq. (17) implies that $\varepsilon = 1$, so that there is no retardation and the gas front recedes with the velocity of the gas particles.

Case B.

In this case the shock occurs in the middle of a region where no free gas is present, so that the velocity of the shock is not determined by Eq. (16). However, the rate $g^o_{\Sigma g}$, at which the gas goes into the oil phase is necessarily zero, and the jump condition (12), reduces to

$$[\bar{R}_s]\overline{\rho_o\left(\underline{v}^o - \underline{v}_\Sigma\right)}^{\bullet} \cdot \underline{n} = 0 \qquad (18)$$

It may be shown that a non-zero jump $([R_s] \neq 0)$, is compatible both with this equation and with the jump condition (4a), if and only if $\underline{v}^o \cdot \underline{n}$ is continuous and

$$\underline{v}_\Sigma \cdot \underline{n} = \underline{v}^o \cdot \underline{n}, \qquad \text{on } \Sigma \qquad (19)$$

Situations in which shocks of these characteristics may occur are discussed in the next section.

Case C.

In this case gas is present on both sides of the shock. Thus the liquid oil is saturated at both sides of the shock. Assuming, as is usually the case, that R_s is a continuous function of p_o,$[R_s] = 0$, because p_o is continuous across Σ (Eq. (6) of Section 2). This implies that

$$\varepsilon = 1 + \delta \frac{\rho_{g-} S_{g-}}{[\rho_g S_g]}$$

A more transparent relation is

$$\underline{v}_\Sigma \cdot \underline{n} = \frac{[S_g \underline{v}^g]}{[S_g]}$$

which can be derived combining Eq. (20) with Eqs. (15). A special case of this equation is the immiscible and incompressible case considered by the classical Buckley-Leverett theory, for which Eq. (21) reduces to the well known relation (see for example [2]):

$$\underline{v}_\Sigma \cdot \underline{n} = \phi^{-1} \frac{[f_g]}{[S_g]} \underline{u}_T \cdot \underline{n} \qquad (22a)$$

where

$$\underline{u}_T = \underline{u}_o + \underline{u}_g \quad \text{and} \quad \underline{u}_g = f_g \underline{u}_T$$

SHOCK GENERATION

In this section we discuss the mechanisms of shock generation for the kinds of shocks that were introduced in last Section.

IN AN UNSATURATED REGION

In a region where the gas phase is present, the oil is necessarily saturated and R_s is uniquely determined by pressure. On the other hand, where the gas phase is absent the liquid oil will usually be undersaturated and R_s can take any value below the saturation curve (Fig. 1). This provides some insight into the initial conditions associated with well-posed problems. When free gas is present, the oil is saturated and the pressure determines R_s so that it is not necessary to include this parameter in the initial conditions. On the other hand, if the oil is undersaturated R_s is not determined by the pressure and must be prescribed as an initial condition. One point which is relevant for our discussions is that the prescribed initial or boundary values of R_s, may be discontinuous. In this case a shock in the unsaturaturated region would be introduced, as it will be seen in what follows.

One property of "oil particles" which move in the interior of a region occupied by undersaturated oil is that they conserve their bubble-point. This property will be used in the sequel.

BUBBLE POINT CONSERVATION PRINCIPLE

In the absence of a gas phase, oil particles conserve their bubble-point.

Proof. When a gas phase is not present the mass exchange term g^{o}_{lg} necessarily vanishes in the governing differential equation (10), and therefore

$$\left(\overline{R}_s\right)_t + \underline{v}^o \cdot \nabla \overline{R}_s = 0 \qquad (23)$$

Thus the "material particle derivative" of R_s vanishes. Clearly this implies that R_s (i.e., the bubble point) remains constant on liquid oil particles.

If the liquid oil is initially undersaturated, R_s is prescribed as part of the initial conditions. If the initial conditions are discontinuous and the liquid oil particles retain their R_s values, then the discontinuity will propagate with velocity $\underline{v}^o$, as required for the jumps, by the mass conservation condition (Case B of Section 3, Eq. (19)). Thus shocks of this kind can be produced by the initial or possibly by the boundary conditions. Later in this section, it will be seen that they can also be generated when an advancing gas-front stops and starts to recede.

AT A GAS FRONT

The bubble-point conservation principle is very restrictive condition and at a gas front shocks are generated when undersaturated particles reach the front and become suddenly saturated. This would happen at an advancing gas front, but not at a gas front that is receding from a region occupied by undersaturated oil, as is explained next.

The evolution of R_s on an oil particle is restricted by the "bubble-point conservation principle". The paths in the R_s - p_o plane described by the values of R_s consist of fragments of the saturation curve or of horizontal segments, only (Fig. 1a). The first ones take place in periods spent by the particle in regions where the gas phase is present, while the latter ones correspond to periods spent by the particle in undersaturated regions where the gas phase is necessarily absent. Thus, if a particle starts at state "n" Fig. 1a, so that it is undersaturated initially, and if it is then depressurized, it moves along a horizontal line towards the left until it reaches the saturation curve. If depressurization of the particle continues, it bubbles and liberates gas. If depressurization is stopped, the free gas is removed and the oil is repressurized, so that the state of the particle in the R_s - p_o plane moves along a horizontal line, this time towards the right. It finally reaches a state such as "n+1" (Fig. 1a).

This path is reversible: we could start at state "n+1" and by successive depressurization and repressurization, reach state "n". The point at which the mixture leaves the saturation curve when it is repressurized depends on the amount of free gas available. In actual reservoirs, this amount of gas is supplied by the gas phase, which in turn is determined by the relative motion of the liquid oil phase with respect to it.

On the other hand, on the R_s-p_o plane the states of an oil particle cannot follow a path such as the one joining states"n" and "n+1" (Fig. 1b) since this would imply that R_s changes without reaching the bubble point. That is, R_s would change when the gas phase is absent and the bubble-point conservation principle would be violated.

At an advancing gas front

At first glance, the previous discussion suggests that in a beta model the only way in which an undersaturated oil particle may become saturated is by depressurization to the bubble point. This would imply that the beta model is a very limited model, especially when considering problems in which the bubble-point varies, since it cannot mimic the processes by which an undersaturated particle of oil receives gas from other particles. However, such limitation is somewhat relaxed by the fact that in a beta model an oil particle may become saturated, in another manner: it may follow a discontinuous path on the $R_s - p_o$ plane, as illustrated in Fig. 1c. This corresponds to an oil particle which is initially undersaturated (point "n") so that the gas phase is absent. At some point the oil particle is reached by a gas front (point SH) and becomes suddenly saturated; under further pressurization R_s moves along the saturation curve. Such a path has a discontinuity at SH and therefore $[R_s] \neq 0$, there, in actual reservoir models. Clearly this corresponds to a discontinuous front or shock. In this case, the shock itself constitutes a mechanism for transferring gas from the gas-phase to the oil-particles. At an advancing gas front, due to the bubble-point conservation principle, a shock of this kind generally will occur even if the initial conditions are continuous. This is because the continuity of the initial values does not prevent oil particles, carrying values of R_s below the saturation value, from reaching the gas-front, where R_s necessarily equals the saturation value. This is the mechanism of shock generation at an advancing gas front.

At a receding gas front

At a front that is receding, on the contrary, R_s is necessarily continuous because the gas phase leaves saturated oil behind it, as it goes away. Since R_s is continuous, the only discontinuous variable is the saturation. Setting $[R_s]$=0 in the first of Eqs. (17) yields $\varepsilon = 1$, which implies $v_\Sigma = v^g$; i.e., the gas front moves together with the gas particles that constitute it. Note that if an advancing gas front changes its sense of motion, thus becoming a receding one, at the point where it stops and starts to recede, a discontinuity of R_s at the oil phase remains. In general, at later times such a shock will be located inside the unsaturated region, since the gas front withdraws from it. According to the results of Section 3, such shocks move with the velocity of the oil-particles. This is a mechanism of generation of the kind of shocks that occur in the undersaturated region and that have been described above in this Section.

A bifurcation mechanism

On the other hand, this analysis also indicates that where an advancing front stops and starts to recede the shock "bifurcates", giving rise to two shocks:one in which the only discontinuous variable is the saturation and the other one in which the only discontinuous variable is the bubble-point. This phenomenon is illustrated in Fig. 2. At an advancing gas front, the bubble-point and the saturation are both discontinuous, so that when the sense of motion of the front changes and it starts to recede, the discontinuities of R_s and S_o coincide, at the point of bifurcation; $x=x_B$, in Fig. 2a. However, as the receding motion of the front progresses, these discontinuities split apart because one moves with the velocity of the oil while the other one moves with the velocity of the gas. This is illustrated in Fig. 2b, where the discontinuity of the bubble-point (R_s) is located at $x_{\Sigma R_s}$, to the left of x_B, while the discontinuity of the saturation is located at x_{Σ_s}, to the right of x_B.

AT REGIONS WHERE FREE GAS IS PRESENT

This kind of shocks are generated by a mechanism that was originally described by Buckley-Leverett [8,9] and further discussed by many authors. They occur when characteristic curves carrying different values of saturation instersect, giving rise to multi-valued solutions which are not physically admissible. A very clear discusion of this process was presented by Sheldon and Cardwell [12]. A recent account, from a present-day perspective, is given in [2].

Note that in a region where free-gas is present, such shocks only develop when capillary pressure is neglected. If capillary pressure is present, the continuity condition of Eq. (6) must be satisfied by both the pressure of the oil and of the gas. This is possible only if the capillary pressure is continuous. This implies the continuity of saturation, since capillary pressure is a continuous function of S_o.

However, other kinds of discontinuities that were discussed above may be generated even if capillary pressure is incorporated in the model. This is because in the other cases considered, the gas phase is not required to satisfy Eq. (6), since the gas pressure is not defined at least at one side of the shock.

SUMMARY AND CONCLUSIONS

Black-oil models do not incorporate molecular diffusion, nor mechanical dispersion. A consequence of such omission is a propensity of such models, to developing shocks. This behavior becomes apparent when these models are applied to problems with variable bubble-point, such as a solution gas-drive, and then several kinds of shocks may develop.

This effect is considerably different to that described by Buckley-Leverett theory and occurs when a gas-front moves into, or recedes from, a region occupied by undersaturated oil. To make our points more clear, the results are placed in the general perspective of an exhaustive classification of shocks that may be found in black-oil model applications; three kinds of shocks are identified. Only one of them is described by Buckley-Leverett theory. In addition, a mechanism of shock bifurcation is also explained.

In summary, shocks are classified according to whether they occur at: 1) a gas region, where gas is present at both sides of the shock, 2) a gas front, and 3) a region occupied by undersaturated oil. Simple expressions are given for the velocities of each kind.

For shocks of type 1), only the gas saturation (S_g) can jump and they are essentially described by Buckley-Leverett theory. In case 3), only the bubble-point (i.e., the solution oil:gas ratio (R_s)) can jump. In case 2), at an advancing gas front, both S_g and R_s are discontinuous, and the conditions at the gas front give rise to a Stefan problem [13]. In addition, when an advancing gas front changes its sense of motion and starts to recede, the shock bifurcates, giving rise to two shocks, one moving with the oil velocity, where only R_s is discontinuous, and the other one with the velocity of the gas, where only S_g jumps.

We feel the results presented here are useful on several counts. Firstly, they contribute to clarify some aspects of black-oil models. For example, a consequence of omitting diffusion and dispersion is the "bubble-point conservation principle": when a gas-phase is not present oil-particles conserve their gas content (bubble-point). Physically, this means that, when a gas-phase is not present, two oil particles cannot exchange dissolved gas, even if they are very close. This property, leads to the preservation of discontinuities (shocks) of the bubble-point.

Another point to be made is that due to the bubble-point conservation principle, the manners in which the oil:gas ratio of an oil particle can vary, are rather restricted. In this respect, it has interest observe that one class of shocks, here presented, constitutes a mechanism by which gas is transferred to oil-particles.

Finally, numerical difficulties occur in numerical models when pathologies such as shocks, are developed; a clear understanding of them is important to design adequate numerical treatments and effectively overcome such difficulties.

REFERENCES

1.-Herrera, I., A. Galindo and R. Camacho, "Shock Modelling in Variable Bubble Point Problems of Petroleum Engineering", Computational Modelling of Free and Moving Boundary Problems, Vol. 1: Fluid Flow, Eds. L.C. Wrobel and C.A. Brebbia, Computational Mechanics Publication & de Gruyter, pp. 399-415, 1991.

2.-Herrera, I., A. Galindo and R. Camacho., Shock Modelling in Petroleum Engineering, Chapter 7 of the book Computational Methods for Free and Moving Boundary Problems in Heat and Fluid Flow, L.C. Wrobel and C.A. Brebbia Eds., Computational Mechanics Publications, pp 143-170, 1993.(Invited Chapter).

3.-Herrera, I., R. Camacho and A. Galindo, "Mechanisms of Shock Generation in Variable Bubble-Point Systems", Computational Modelling of Free and Moving Boundary Problems, Vol. 2, L.C. Wrobel and C.A. Brebbia Eds., Computational Mechanics Publicactions, Southampton & Boston, pp 435-453, 1993 (Invited paper).

4.-Aziz, K., and Settari, A., "Petroleum Reservoir Simulation", Ed. Applied Science Publishers Ltd, London, 1979.

5.-Glimm, J., E. Isaacson, D. Marchesin and O. McBryan, Front Tracking for Hyperbolic Systems, Adv.Appl.Math. 2, 91-119, 1981.

6.-Colella, P., P. Concus and J. Sethian, Some Numerical Methods for Discontinuous Flows in Porous Media, in The Mathematics of Reservoir Simulation, R.E. Ewing, Ed., Frontiers in Applied

Mathematics Vol. 1, SIAM, Philadelphia, 161-186, 1983.

7.-Bratvedt F., K. Bratvedt, C.F. Buchholz, T. Gimse, H. Holden and N.H. Riseboro, Front Tracking for Groundwater Simulations, Computational Methods in Water Resources X, Vol. I: Numerical Methods in Water Resources, T.F. Russell et al. Eds., CM Publications, Elsevier Applied Science, 97-104, 1992.

8.-Leverett, M.C.,"Capillary Behaviour in Porous Solids", Trans. AIME, (1941), 142, 152.

9.-Buckley, S.E. and Leverett, M.C."Mechanics of Fluid displacement in sands". Trans., AIME (1942) 146, 107.

10.-Welge, H.J.,"A Simplified Method for Computing Oil Recovery by Gas or Water Drive", Petroleum Trans. AIME, (1952), 195, pp 91-98.

11.-Cardwell, W.T.,"The Meaning of the Triple Value in Noncapillary Buckley-Leverett Theory", T.P. 808, Trans. AIME, 1959, 216, pp 271-276.

12.-Sheldon, J. W. and Cardwell, W. T., "One-Dimensional, Incompressible, Noncapillary, Two-Phase Fluid in a Porous Medium", Petroleum Trans. AIME, 1959, 216, pp 290-296.

13.-Crank, J. (Ed.), "Free and Moving boundary problems", Clarendon Press, Oxford, 425p, 1984.

14.-Raghavan, R., "Pressure Transient Analysis of a Vertically Fractured Well Produced by Solution Gas Drive", Soc. Pet. Eng. J. (1977), 369-376.

15.-Camacho-V., R.G. and Raghavan, R., "Some Theoretical Results Useful in Analyzing Well Performance Under Solution Gas Drive", Paper SPE 16580.

16.-Camacho-V., R.G. and Raghavan, R., "Boundary-Dominated Flow in Solution Gas-Drive Reservoirs", SPE Reservoir Engineering, Dec. 1989.

FIGURE CAPTIONS

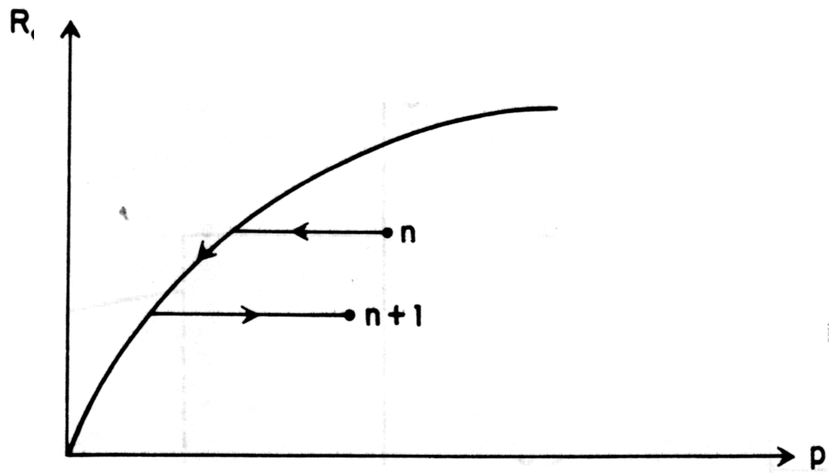
FIGURE 1.- Paths in the R -p plane.

FIGURE 2.- The bifurcation mechanism.

a) The bifurcation point at x_B .

b) The two shocks, after bifurcation.

□



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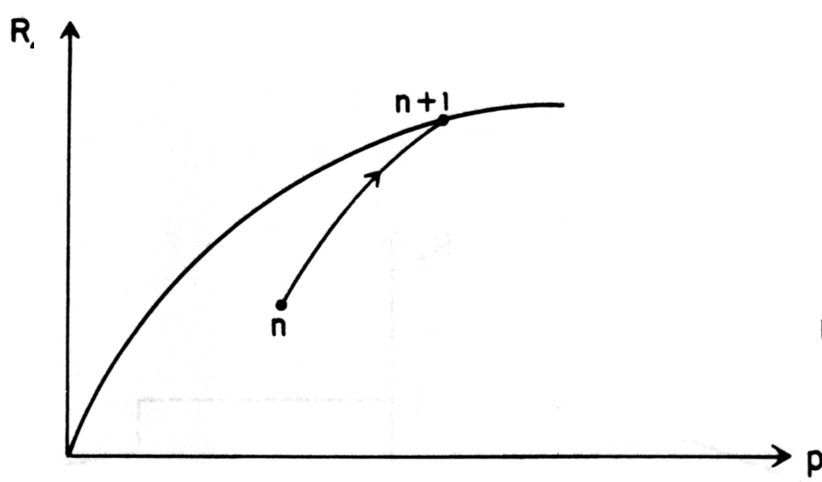

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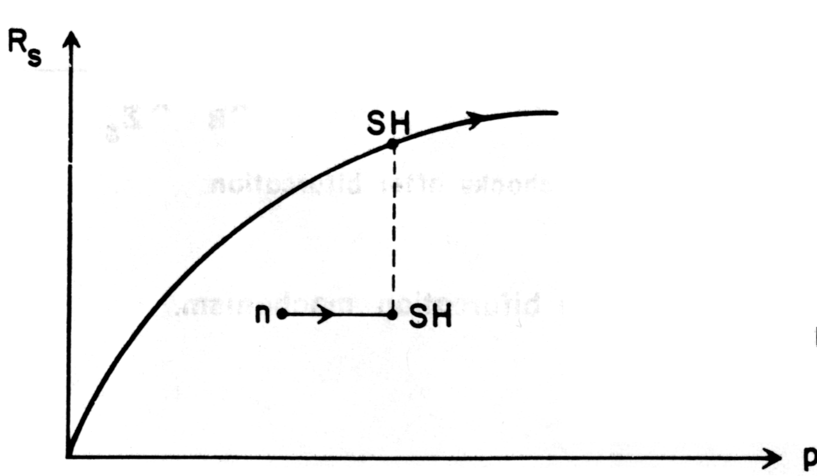
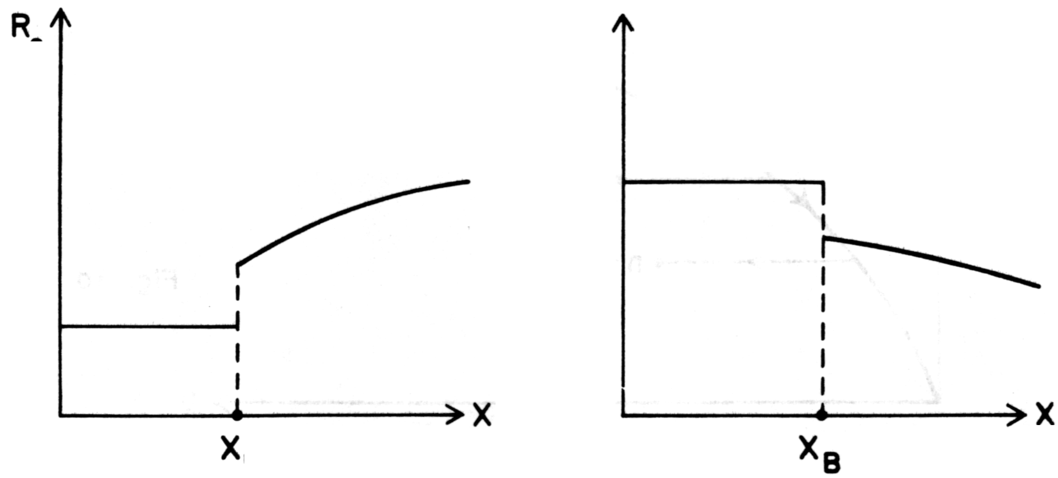


Fig.

Figure Paths in the R_S p plane.



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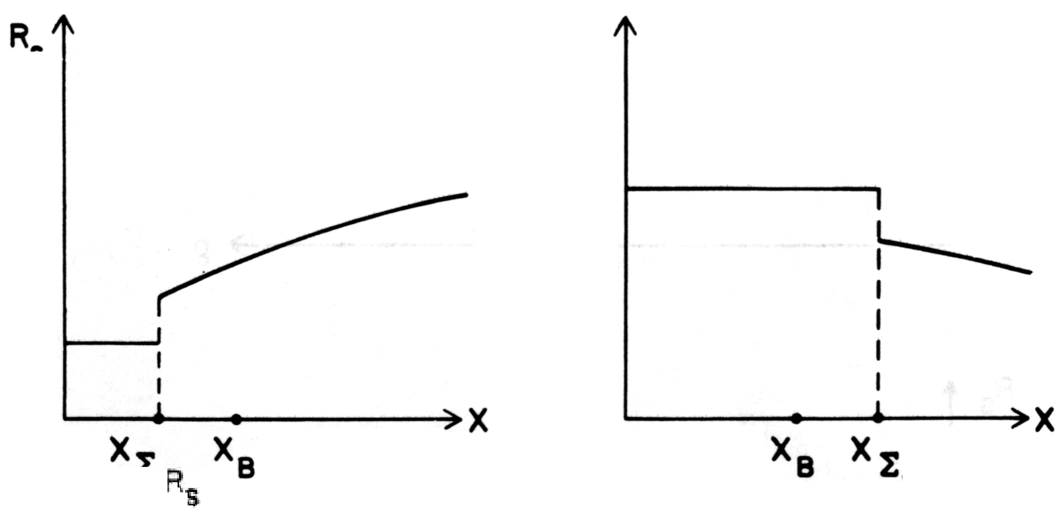

b. Th two sh cks ter bifurcation

Figure Th ifurcation mechan sm