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A consistent black-oil model with variable bubble-point

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ABSTRACT

The 'traditional' approach to variable bubble-point problems, using black-oil models, is not consistent, because it violates the 'bubble-point conservation law'. Here, a consistent procedure for dealing with such problems is given. It incorporates shocks, in which the bubble-point is discontinuous. Three kinds of shocks and a bifurcation mechanism, are required. This 'consistent' approach is applied to specific examples, and results compared with those of the 'traditional one'. The conclusion is that the 'traditonal' approach generally yields large errors for the production rates and other quantities of interest in the oil industry.

1 INTRODUCTION

It has been shown that the traditional black-oil model approach to variable bubble-point problems [1-4], is inconsistent [5,6], and that in this manner large errors in the production rates and other quantities of interest in the oil industry, are generated [7].

A consistent treatment of variable bubble-point problems has been presented by the authors. It requires the introduction of shocks in which the bubble-point -i.e., the solution gas-oil ratio: R_s- is discontinuous. The following classes of shocks have to be modeled [5,6]: shocks in which the only discontinuous variable is saturation -generally, occurring in regions where free-gas is present or at a receding gas-front-, shocks in which the only discontinuous variable is R_s -occurring in regions where free-gas is absent-, and shocks in which both the saturation and R_s are discontinuous -occurring at an advancing gas front.

In addition, a bifurcation occurs when an advancing gas-front -which carries a double discontinuity: discontinuous saturation and discontinuous bubble-point- changes its sense of motion and starts to recede; then, the two discontinuities start to move with different velocities and in this manner, they separate. In such case, the bubble-point discontinuity moves with the oil-particle velocity, while the saturation discontinuity moves with the gas-particle velocity. Shocks occurring in regions where free-gas is present behave in a manner similar to that described by Buckley-Leverett theory [8]. However, the other kinds of shocks have a very different character; in particular, they may develop even if capillary pressure is incorporated in the model. A brief presentation of the above, is given in what follows.

2 INCONSISTENCY OF THE TRADITIONAL APPROACH

We consider a "black-oil" or "beta" model, consisting of two phases, liquid oil and gas -whose Darcy velocities are denoted by $\underline{u}_o$ and $\underline{u}_g$, respectively [5,6]. As usual, no physical diffusion is included.

When dealing with variable bubble-point problems, in general, in which free gas may, or may not, coexist with liquid oil, the region of definition of the problem may be decomposed into three parts [5,6]: one in which free-gas is present (the "gas-region"), another one in which free-gas is absent (the "oil-region"), and a gas-front which limits these two regions. To be specific, at the gas-front, the unit normal vector ($\underline{n}$) will be taken as pointing towards the gas region. When considering discontinuities across a surface Σ, the velocity of Σ will be $\underline{v}_\Sigma$, and the unit normal vector will be taken pointing towards the positive side. In particular, at the gas-front the gas-side will be the positive side Also, square brackets stand for the "jump" of a function; thus, for example: $[R_s]= R^+_s-R^-_s$.

When applying black-oil models to variable-bubble-point problems, it is frequently assumed that the bubble-point pressure may vary inside the -undersaturated- 'oil-region' [1-4]. However, such assumption is incorrect, because it violates the "bubble-point conservation law" [5,6], which states that in the absence of a gas phase, oil-particles conserve their bubble-point; i.e., [5,6]:

$$(R_s)_t + \underline{v}_o \bullet \mathrm{grad}(R_s) = 0 \qquad (1)$$

In words: "the material oil-particle derivative of R_s vanishes". Clearly, this implies that R_s (i.e., the bubble-point), remains constant on liquid oil particles.

The bubble-point conservation law is a very restrictive condition; physically it means that, when a gas-phase is not present, two oil-particles cannot exchange dissolved gas, even if they are very close. In particular, this property leads to preservation of discontinuities of the bubble-point.

Due to the "bubble-point conservation law", the paths that an oil-particle can describe on the R_s -p_o plane, consist of fragments of the saturation curve or of horizontal segments, only (Fig. 1a). The first ones take place in periods spent by the particle in regions where the gas-phase is present, while the latter ones correspond to periods spent by the particle in undersaturated regions, where the gas-phase is absent. Thus, if a particle starts at state "n" (Fig. 1a), so that it is undersaturated initially, and it is then depressurized, it moves along a horizontal line towards the left, until it reaches the saturation curve. If depressurization of the particle continues, it bubbles and liberates gas. At such point, the state of the oil-particle lies on the saturation curve. If it is represurized, at first it will move along the saturation curve, until all the free-gas is removed, and at such point it will start to move along a horizontal line, towards the right, leaving the saturation curve, until it finally reaches a state such as "n+1" (Fig. 1a). This path is reversible: we could start at state "n+1" and by successive depressurization and repressurization, reach state "n". The point at which the saturation curve is left by the oil-particle, under repressurization, depends on the amount of free-gas available. In actual reservoirs, this amount of gas is supplied by the gas-phase, and it is determined by the relative motion of the oil-phase with respect to the gas-phase.

On the other hand, on the R_s -p_o plane, the states of an oil-particle cannot follow a path such as the one joining states "n" and "n+1" (Fig. 1b), since this would imply that R_s changes without reaching the bubble-point. That is, R_s would change when the gas-phase is absent and the bubble-point conservation law would be violated. However, in the 'traditional' approach to variable bubble-point reservoirs, such paths on the R_s-p_o plane are admitted [1]. This is the inconsistency of the traditional approach, we refer to.

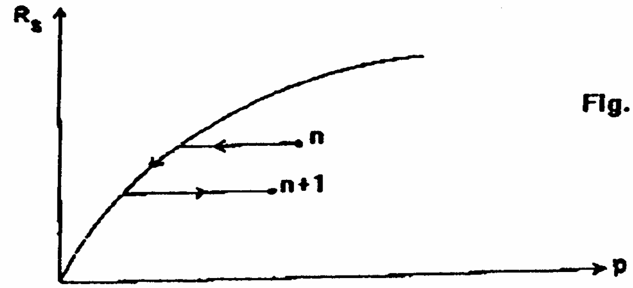

Fig. 1a

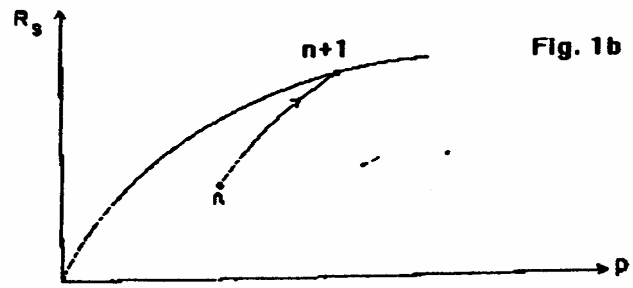

Fig. 1b

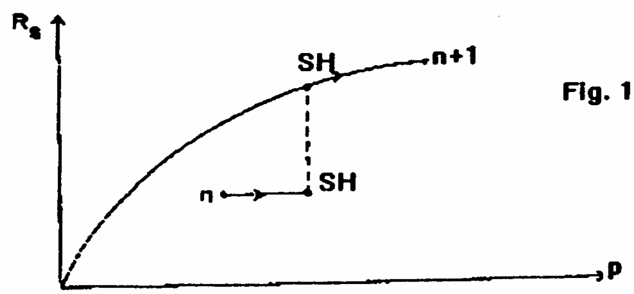

Fig. 1c

3 A CONSISTENT APPROACH

At first glance, the previous discussion suggests that in a black-oil model the only way in which an undersaturated oil-particle may become saturated, is by depressurization to the bubble-point. This, however, is not correct, because in a black-oil model an oil-particle may become saturated, in another manner: it may follow a discontinuous path on the R_s-p_o plane, such as SH-SH', in Fig. 1c. This corresponds to an oil-particle which is initially undersaturated (point "n"), so that the gas-phase is absent. At some point the oil-particle is reached by a gas-front (point SH) and becomes suddenly saturated (point SH'); under further pressurization R_s moves along the saturation curve. Such a path has the discontinuity SH-SH', and therefore $[R_s] \neq 0$, there. In actual reservoir models, this corresponds to a discontinuity of R_s , at the gas-front. The physical implication is that the gas transference, at the gas-front, from the gas-phase into the liquid-oil-phase, is so intense, that it gives rise to a discontinuity in R_s .

Thus, a consistent formulation of black-oil models requires the incorporation of shocks in which not only the saturation, but also the dissolved gas-oil ratio R_s, may be discontinuous. The remaining of this Section is devoted to present an exhaustive description of the schoks needed (for a more complete presentation, see [5,6]). In particular, the kind of shocks that occur in each one of the three parts in which the region of study has been divided, will be discussed.

AT A GAS-FRONT

Two situations must be distinguished.

At an advancing gas front

According to the previous discussion when oil particles carrying values of R_s below the saturation value reach the gas-front -where R_s necessarily equals the saturation value-, a discontinuity will be produced (segment SH-SH′, of Fig.1c).In general at such a shock there are two discontinuous variables: $[R_s] \neq 0$ and $[S_o] \neq 0$.

At a receding gas front

At a front that is receding, on the contrary, R_s is necessarily continuous, because the gas phase leaves saturated oil behind it, as it goes away. In this case, the only discontinuous variable is the saturation.

IN THE OIL-REGION

In that region no free-gas is present and the oil saturation equals to one, necessarily. Thus, the only possible discontinuous variable is R_s .In contrast to the "gas-region", where oil is necessarily saturated and R_s is uniquely determined by pressure, when the gas-phase is absent the liquid oil will usually be undersaturated and R_s can take any value below the saturation curve (Fig. 1a). Thus, a discontinuity in R_s can be introduced through the initial or boundary conditions.

AT THE GAS-REGION

In this region, R_s is a continuous function of position. Thus, the only possible discontinuous variable is saturation. Such shocks were originally described by Buckley and Leverett, and further discussed by many authors. A recent account, from a present-day perspective, is given in [8].

Note that, as it is generally recognized, such shocks are developed only when capillary pressure is neglected. However, the other kinds of discontinuities, that have been discussed above, may occur even if capillary pressure is incorporated in the model.

4 SHOCK VELOCITIES

In this Section, a summary of the velocities for each of the shocks just described, that were derived in [5,6], is given.

AT A GAS-FRONT

According to [5,6], the relative velocity of the front, with respect to the oil, is given by:

$$(\underline{v}_\Sigma - \underline{v}^+_o)\bullet\underline{n} = \varepsilon(\underline{v}_g - \underline{v}^+_o)\bullet\underline{n}, \qquad (2)$$

where the 'retardation factor' ε is:

$$\varepsilon = \cfrac{1}{1+[R_s]\cfrac{\bar{\rho}^+_o S^+_o}{\rho^-_g S_g}}, \qquad (3)$$

Observe that the $0<\varepsilon<1$, and therefore ε can indeed be interpreted as a retardation factor, -of the shock motion with respect to the gas. When the gas-front is advancing into the oil region, $[\bar{R}_s] \equiv \bar{R}^+_s - \bar{R}^-_s > 0$, so that $\varepsilon<1$, and the retardation effect, of the motion of the shock respect to the gas, is present. On the contrary, if the front is receding, $[R_s]=0$, $\varepsilon=1$ the retardation effect is absent, and the shock moves with the gas velocity.

IN THE OIL-REGION

In this case the only possible discontinuous variable is R_s, since $S_o=1$, at both sides of the shock, and it has been shown [5,6] that:

$$\underline{v}_\Sigma . \underline{n} = \underline{v}_o \bullet \underline{n}, \quad \text{on } \Sigma. \qquad (4)$$

AT THE GAS-REGION

In this region

$$\underline{v}_\Sigma \bullet \underline{n} = \frac{[\underline{u}_g] \bullet \underline{n}}{\phi [S_g]} \qquad (5)$$

Of course, a special case of this equation is the immiscible and incompressible case considered by the classical Buckley-Leverett theory, for which [8]:

$$\underline{v}_\Sigma \bullet \underline{n} = \phi^{-1} \frac{[f_g]}{[S_g]} \underline{u}_T \bullet \underline{n} \qquad (6)$$

where $\underline{u}_T$ is the total Darcy velocity.

5 A SHOCK BIFURCATION

At a point where a gas-front, advancing into a region of undersaturated gas, changes its sense of motion, thus becoming a receding one, the double discontinuity ($[R_s] \neq 0$ & $[S_o] \neq 0$) of the front `bifurcates'[5,6], giving rise to two shocks: one in which the only discontinuous variable is the saturation -moving with the gas velocity- and the other one in which the only discontinuous variable is the bubble-point -moving with the oil velocity.

6 A NUMERICAL EXAMPLE

The purpose of this example is to illustrate the magnitude of numerical errors introduced by the application of a `traditional' black-oil models [6-9], which have been shown to be <u>inconsistent</u>, to variable bubble-point problems. Thus, the example will be treated by two methods: one in which shocks are explicitly incorporated using the authors' theory, and the other one will be a `traditional' formulation [1-4], in which shocks are not incorporated. Then, comparisons of the results of these methods will be made.

In this example the saturation R_s -p curve of Fig. 2, for which R_s is pressure dependent everywhere, will be used. The initial conditions for this one-dimensional problem, consist of a left-region of undersaturated oil and a right-region containing a gas-phase, where the oil is saturated. These two regions are separated by a gas-front. Production is obtained from the left-boundary at a constant pressure, while gas is injected on the right-boundary, at a variable rate: $v_g = 10(t-30)$.

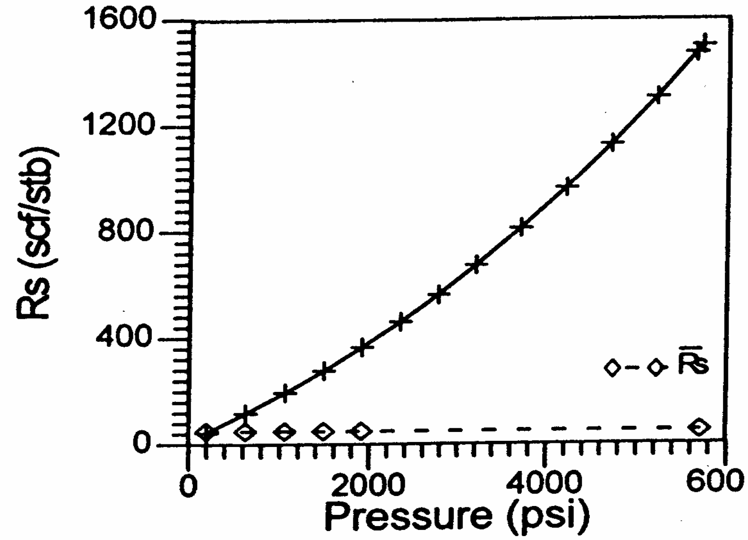

Fig. 2 Curve of gas in solution.

Therefore, at the gas-front ($x_{\Sigma SR}$), when it is advancing into the region of undersaturated oil, two discontinuities coexist -one of saturation (S_o) and other one of the solution gas:oil ratio (R_s). Initially, the gas-front moves towards the left with variable velocity, which is not given by a simple closed expession (see Fig. 3), because the retardation factor ε, as given by Eq. (3), is fully dependent on the detailed shape of the R_s -p curve, illustrated in Fig. 2. At $t=t_B$ =30 days, the sense of motion changes and the shock bifurcates, since the discontinuities start to recede with different velocities. The velocity of the front is non-constant and it is not possible to give a simple, closed expression for it. The velocity $v_{\Sigma RS}$, of the advancing gas-front, and the retardation factor ε, are shown in Fig. 4. Both are strongly influenced by the detailed shape of the saturation R_s -p_o curve.

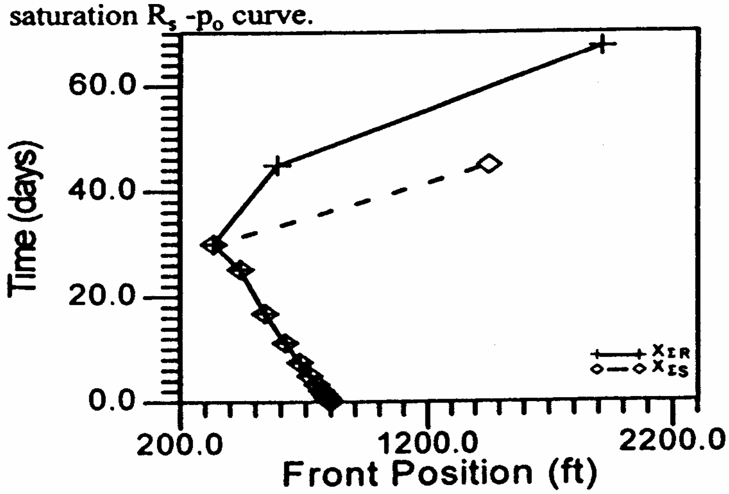

Fig.3 Front Position for Example, with bifurcation tim

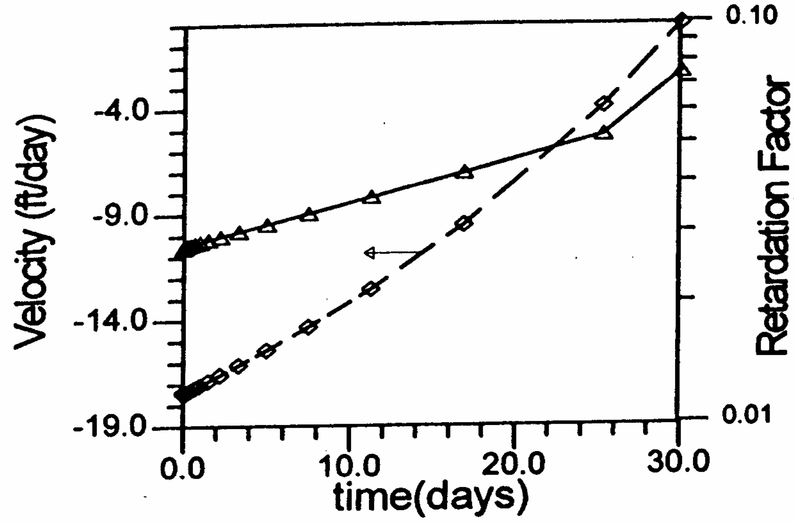

Fig. 4 Front velocity and Retardation Factor ($t < t_B$)

In order to establish the significance of using a 'consistent' black-oil model when treating variable bubble-point problems, the example that has just been described, was also treated using the 'traditional numerical formulation' [1-4]. When a 'traditional formulation', in which no discontinuity in R_s is incorporated, the saturation front advances towards the undersaturated region, much faster than it should, as illustrated in Fig. 5. Thus, in about three days the saturation front reaches the left-boundary; this is about one tenth of the bifurcation time t_B of our 'consistent' black-oil model, in which the gas-front begins to recede without reaching the left-boundary and, therefore, it never reaches this boundary. Obvious consequences of this drastic difference in the front velocities, when the inconsistent 'traditional' black-oil model is applied, are big inaccuracies in the evaluation of gas production at the boundary and other quantities of interest for the oil industry.

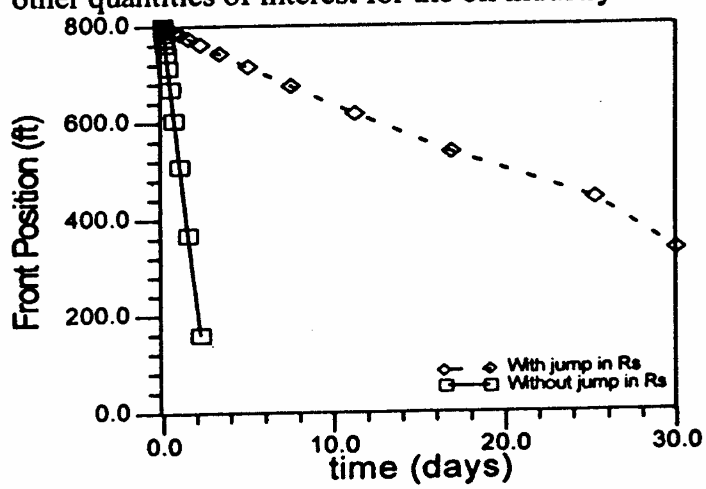

Fig. 5 Comparison of front position, with and without jum

7 DISCUSSION AND CONCLUSIONS

In usual formulations of black-oil models neither molecular difusion nor mechanical dispersion is included. A consequence of such omission is the 'bubble-point conservation law' [5,6], according to which when a gas-phase is not present, oil-particles conserve their gas content (bubble-point). However, when applying black-oil models to variable bubble-point problems, it is customary to assume that the bubble-point pressure may vary inside the undersaturated region [1-4], and this is an inconsistency of the 'traditional' approach to variable bubble-point problems.

A consistent black-oil model formulation, of variable bubble-point problems, requires incorporating shocks in which the solution gas:oil ratio R_s is discontinuous. Generally, three kinds of shocks and a bifurcations mechanism, are needed [5,6]. Such shocks can be classified according to whether they occur at: 1) a gas-front, 2) a region occupied by undersaturated oil, and 3) a gas-region. Simple expressions are given for the velocities of each one of them. For shocks of type 3), only the saturation can jump and they are essentially described by Buckely-Leverett theory. In case 2), only the bubble-point (i.e., the solution oil:gas ratio (R_s)) can jump. In case 1), at an advancing gas-front, both S_o and R_s are discontinuous, but if the gas-front is receding, then only the saturation can jump. Except for shocks of type 3), all others may occur even if capillary forces are present [5,6]. In addition, when an advancing gas-front changes its sense of motion and starts to recede, the shock bifurcates, giving rise to two shocks, one moving with the oil velocity, where only R_s is discontinuous, and the other one with the velocity of the gas, where only S_o jumps. In addition, it has been shown that numerical predictions made using the 'traditional' -inconsistent- approach [1-4], in which no discontinuities in R_s are incorporated, may differ drastically from those made using a 'consistent approach', here explained [5,7].

The "bubble-point conservation law", is a very severe restriction to the manners in which the solution gas: oil ratio of an oil-particle can vary. Physically, it means that, when a gas-phase is not present, two oil particles cannot exchange dissolved gas, even if they are very close. This property, leads to preservation of discontinuities (shocks) of the bubble-point and explains why shocks of types 1) and 2), occur even if capillary forces are included in the model. Although capillary forces are frequently perceived as diffusive -and in some instances such perception is justified-, the "bubble-point conservation law" holds even in the presence of such forces. On the other hand, when either molecular diffusion, mechanical dispersion, or both, are included in the models, the "bubble-point conservation law" ceases to be valid.

Of course in the physical reality, diffusion mechanisms are always present, even if in some cases they may be rather weak. On the other hand, mathematical models are approximations, in which always not all the complexity of the physical reality is incorporated. For a 'modeler', a wise strategy is to incorporate only those processes which are relevant, at the level of accuracy required. If in some application, diffusion is not significant it is correct to neglect it and apply the black-oil model, in a consistent manner - incorporating shocks-, as it has been explained here and in previous papers [5,6]. However, if on the contrary, diffusion processes are relevant, the model must be modified and diffusion processes -either molecular diffusion, mechanical dispersion, or both- must be incorporated explicitly. What must not be done, however, is to apply an incosistent model, because numerical results obtained using incsistent models, are unwarranted. Indeed, for example, the velocity of an advancing gas-front predicted by the inconsistent traditional approach, is the same as the velocity of the gas-phase. However, such velocity when it is predicted by a consistent approach, differs considerably from that of the gas-phase.

One may expect that the consistent approach to non-diffusive black-oil modeling be the limit, as diffusion goes to zero, of diffusive modeling. And, in this respect, it would be important to establish the range where diffusion mechanisms can be neglected. Work oriented to elucidate such matters is underway.

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FIGURE CAPTIONS

FIGURE 1.-Paths in R_s-p_o plane

FIGURE 2.-Curve of gas in solution.

FIGURE 3.-Front Position for Example, with bifurcation time

FIGURE 4.-Front velocity and Retardation Factor ($t<t_B$)

FIGURE 5.-Comparison of front position, with and without jump