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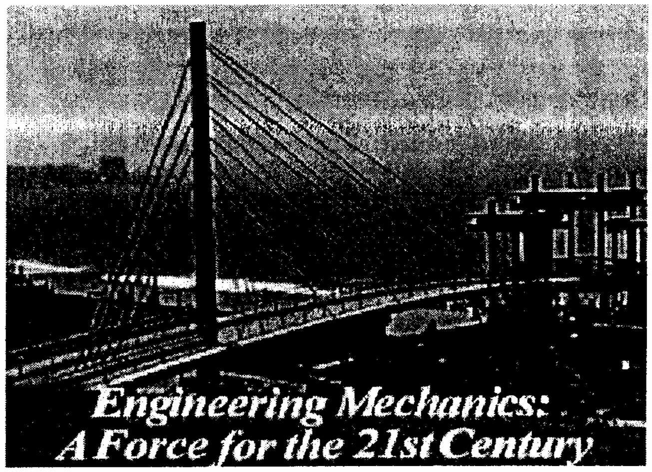

Trefftz-Herrera Formulation of Domain Decomposition

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Abstract

Domain decomposition methods have been extensively studied, specially during the last decade, as a very effective tool for parallelizing models of continuous (macroscopic) systems. In general, there are two approaches to develop domain decomposition methods: One starts with the discretized version of the model and the other one with the partial differential equations, before they are discretized. A very general formulation of this latter approach, which is applicable to any partial differential equation or system of such equations -symmetric or non-symmetric with coefficients that may be discontinuous-, is Trefftz-Herrera domain decomposition, recently presented by the author. Here this method is applied to a kind of elliptic equations occuring in transport problems.

Introduction

Domain decomposition techniques have received much attention in recent years. They constitute a natural route to parallelism. Using them it is possible to transform large discrete systems into smaller ones. Also, domains of irregular shape can be decomposed into regular subdomains in which tensor-product discretizations can be applied. In addition, domain decomposition techniques are quite suitable for carrying out grid refinements in regions where they are required, as where the coefficient variability is high.

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For elliptic problems, domain decomposition methodologies are well developed. In many instances time dependent problems of parabolic type can be treated in the same manner, because for usual time discretizations, they give rise to an elliptic problem at each time-step. Using this approach domain decomposition methods have been applied to transport problems (Herrera et al. 1994; Guarnaccia et al. 1994).

An approach to domain decomposition -Trefftz-Herrera Domain Decomposition- very general, applicable to any partial differential equation or system of such equations -symmetric or non-symmetric with coefficients that may be discontinuous-, which was recently introduced by the author (Herrera 1995, 1997), is here applied to derive a domain decomposition procedure for any elliptic differential equation of second order, which is the kind of operator occurring when transport problem when a time-stepppping procedure is used (Herrera et al. 1994; Guarnaccia et al. 1994).

Domain Decomposition for Elliptic Equations

Consider the most general elliptic equation of second order:

$$\mathcal{L} u \equiv - \nabla \cdot (\underline{\underline{a}} \cdot \nabla u) + \nabla \cdot (\underline{b} u) + c u = f_\Omega \tag{1}$$

Let trial and test functions be continuous across the internal boundaries Σ, (see Figure 1) with first derivatives possibly discontinuous. For definiteness take Dirichlet boundary conditions:

$$u = u_\partial \; ; \quad \text{on } \partial\Omega \tag{2}$$

and jump conditions

$$\left[\frac{\partial u}{\partial n}\right] = \left[\frac{\partial u_\Sigma}{\partial n}\right]; \quad \text{on } \Sigma \tag{3}$$

where u_Σ is a given function.

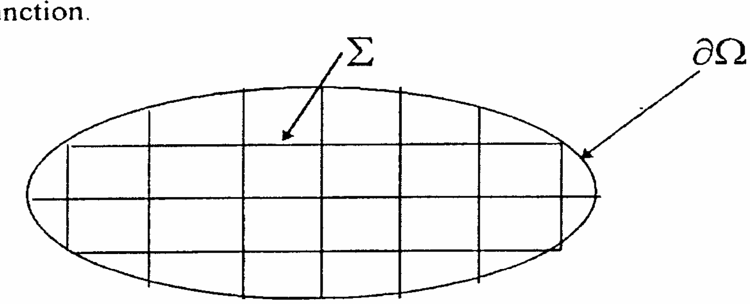

Figure 1: The Region Ω

A subset of specialized weighting functions will be applied. They satisfy

$$\mathcal{L}^* w \equiv - \nabla \cdot (\underline{\underline{a}} \cdot \nabla w) - \underline{b} \cdot \nabla w + c\, w = 0; \quad \text{in } \Omega \tag{4a}$$

and

$$w = 0\,; \quad \text{on } \partial\Omega \tag{4b}$$

The domain decomposition procedure is characterized by (Herrera 1998):

$$- \int_\Sigma a_n \, \hat{u} \left[\frac{\partial w}{\partial n} \right] dx = \int_\Sigma a_n \, w \left[\frac{\partial (u_\Sigma - u_p)}{\partial n} \right] dx\,; \tag{5}$$

where $a_n = \underline{n} \cdot \underline{\underline{a}} \cdot \underline{n}$. The procedure is a domain decomposition method because the weighting functions, may be constructed locally. For example, consider the rectangle of Fig. 2, divided into

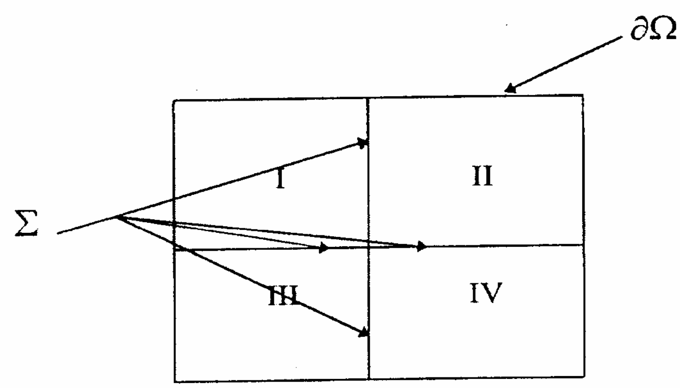

Figure 2: Domain Decomposition of the Example

four subregions. If the value of w is prescribed on Σ, we have well posed problems defined on each one of the subregions I to IV, because Eqs. (4) must be satisfied. Thus, w can be constructed in each one of the subregions separately.

References

Herrera, I., Guarnaccia, J., and Pinder, G.F. (1994). "Domain decomposition method for collocation procedures." Computational Methods in Water Resources X, Vol. 1, Eds. A. Peters., et al., Kluwer Academic Publishers, International Conference on Computational Methods in Water Resources, Heidelberg, 273-280.

Guarnaccia, J., Herrera, I., and Pinder, G.F. (1994). "Solution of flow and transport problems by a combination of collocation and domain decomposition procedures." Computational Methods in Water Resource X, Vol. 1, Eds. A. Peters., et al., Kluwer Academic Publishers, International Conference on Computational Methods in Water Resources, Heidelberg, 265-272.

Herrera, I. (1995). "Trefftz-Herrera domain decomposition." Special Volume on Trefftz Method: 70 Years Anniversary: Advances in Engineering Software, 24, 43-56.

Herrera, I. (1997). "Trefftz-Herrera method." Computer Assisted Mechanics and Engineering Sciences, Polish Academy of Sciences, 4(3/4), 369-382.

Herrera, I. (1998). "Unified approach to domain decomposition." Proceedings of the Fourth World Congress on Computational Mechanics, New Trends and Applications, E. Oñate and S.R. Idelsohn Eds. Barcelona, Spain. (In press).