Unified formulation of domain decomposition based on Trefftz-Herrera method

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The author's Algebraic Theory of Boundary Value Problems [1, 2] has permitted systematizing Trefftz Method and expanding its scope. The concept of TH-completness has played a key roll for such developments [3]. A generalized version of Trefftz Method, known as LAM (Localized Adjoint Method), has been quite successful in numerical applications [4, 5]. Here, this theory is used to present a formulation of domain decomposition methods possessing great generality [6, 7]. The resulting formulation includes any partial differential equation which is linear, or system of such equations. In particular, equations with discontinuous coefficients are included.

Boundary value problems, with prescribed jumps at internal boundaries, are formulated variationaly, in terms of the data of the problem, in the form

$$(P - B - J)u = f - g - j$$

When use is made of Green-Herrera formula ($P - B - J = Q^\star - C^\star - K^\star$), this becomes the variational formulation in terms of the sought information:

$$(Q^\star - C^\star - K^\star)u = f - g - j$$

Here P, B, J, Q, C and K are bilinear functionals while f, g, j are linear functionals, whose explicit expressions are given in the theory. In particular, K is related to Poincaré-Steklov operators. The above variational formulations imply a kind of operator extensions whose relation with the theory of distributions was explained in [8].

For functions w satisfying $Qw = 0$ and $Cw = 0$, this equation becomes

$$-\langle K^\star u, w\rangle = \langle f - g - j, w\rangle$$

This is the general domain decomposition formulation, we are referring to. In addition the bilinear functional $\langle K^\star u, w\rangle$ is shown to be positive definite when the differential operators are positive definite. Specific applications to elliptic equations of second order, mixed methods and ellasticity, are presented. Application of conjugate gradient for the resulting system is explained and also a very general version of Quarteroni method is derived.

References

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