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A dual-porosity approach to simulate groundwater flow in fractured porous media: application in the Itxina karstic aquifer, Spain

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Abstract Mathematical models are commonly used in groundwater studies as an attempt to represent processes through mathematical equations. In the case of fractured porous media, and especially when karstification processes occur, fractures contribute as a secondary porosity, adding to the original one. This makes the comprehension of the phenomenon and the characterization of the groundwater flow difficult. This is the case of the karst aquifer of Itxina, which presents a high degree of karstification, with particular emphasis on the exuberance and wealth of the exokarstic forms and the development of the underground cavities. The aim of this paper is, first, to present the conceptualization of the Itxina aquifer. The second part treats the modelling of the flows in the aquifer using the TRAFRAP model (TRAnsport in FRActured Porous media with water table boundary conditions, Huyakorn *et al.*, 1994). It is a two-dimensional finite element model designed to simulate groundwater flow in a steady-state or transient form. Simulation results demonstrate the complicated nature of the karst system. With the different simulations and variations to the inputs (recharge), the system was characterized as a highly fractured system with a rapid response to events.

Un enfoque de pososidad dual para simulacion de flujo de aguas subterraneas en medios porosos fracturados: aplicacion en el acuifero kastico de Itxina, España

Resumen Los modelos matemáticos son ampliamente utilizados en el estudio de las aguas subterráneas, constituyendo elementos de representación de diferentes procesos por medio de ecuaciones matemáticas. Así, en el caso de los medios porosos fracturados y, muy en particular, en los que intervienen procesos de kárstificación, las fracturas contribuyen con una porosidad secundaria a la ya existente, introduciendo elementos que dificultan la comprensión del fenómeno y la caracterización del flujo subterráneo. Este es el caso del acuífero kárstico de Itxina, que presenta un alto grado de kárstificación, donde se destaca la exuberancia y riqueza de las formas exokársticas y el desarrollo de las cavidades subterráneas. Bajo estas especiales circunstancias, el objetivo de este artículo es, en una primera parte, presentar la conceptualización del acuífero de Itxina. En una segunda parte, se presenta la modelación de flujo, usando para ello el modelo

TRAFRAP (Huyakorn *et al.*, 1994), basado en el esquema de elemento finito que permite simular el flujo en estado estacionario y transitorio. Los resultados de la simulación demuestran la complicada naturaleza del sistema kárstico. Con base en las diferentes simulaciones y variaciones de los impulsos (recarga), el sistema ha sido caracterizado como altamente fracturado con una rápida respuesta a las impulsiones.

INTRODUCTION

Numerical simulation of flow processes in karstic aquifers is a formidable task, due to the often complex geological and hydrological characteristics of such formations. The specific geometry and other characteristics of the fracture system are generally not known, so it is not possible to explicitly model individual fractures or individual matrix blocks. In karst aquifers, besides the low hydraulic conductivity common to fracture networks with a spacing of a few metres, there is a high hydraulic conductivity channel network with kilometre-wide intervals, which is well connected to a small and narrow discharge area, the karst spring. According to Kiraly *et al.* (1995), the duality of karst aquifers is a direct consequence of this structure: (a) duality of the infiltration processes (diffuse or slow infiltration into the low hydraulic conductivity volume, concentrated or rapid infiltration into the channel network); (b) duality of the groundwater flow field (low flow velocities in the fractured volume, high flow velocities in the channel network); and (c) duality of the discharge conditions (diffuse seepage from the low hydraulic conductivity volume, concentrated discharge from the channel network, at the karst spring).

A number of mathematical models describing groundwater flow and solute transport in fractured porous media have been developed in the past. One classical approach is to consider a fractured porous medium as a single continuum or equivalent porous medium in which the point-to-point spatial variations in the hydrogeological properties of the rock mass are averaged over a representative elementary volume (REV) in order to define bulk macroscopic values (Bear, 1972). Long *et al.* (1982), Berkowitz *et al.* (1988), and Schwartz & Smith (1988), among others, have studied the applicability of this approach in the context of groundwater flow and dissolved solute transport in fractured geological materials under saturated conditions.

A large number of models using the two-domain or multi-domain concept have been used to describe water flow and/or solute transport in macropores soils (e.g. Hoogmoed & Bouma, 1980; Beven & Germann, 1982; Davidson, 1985; Bruggeman & Mostaghimi, 1991), unsaturated fractured rocks (Berkowitz *et al.*, 1988; Dudley *et al.*, 1988), and fissured groundwater systems (Barenblatt *et al.*, 1960; Bibby, 1981). Compartment models have been suggested for flow in fractured reservoirs and solute transport in structured soils (Coats & Smith, 1964). Several authors also assumed a specific geometry of the macropores or fractures for water flow (Wang & Narasimhan, 1985; Pruess *et al.*, 1990) or solute transport (Neretnieks & Rasmuson, 1984; van Genuchten & Dalton, 1986).

Up to now, only a few investigations have been developed in this field. Most of the studies are ideal simulations which show only few relationships with the natural system. Consequently, the first objective of this paper is to give a realistic response of a karst aquifer (submitted to different drives), the Itxina aquifer, so as to conceptualize it. A second objective is to use this analysis to establish the dual porosity model for the same system flow. A series of simulations shows how to predict the response of the system to the recharge variation (infiltration).

FLUID FLOW MODEL

The model used is the TRAFRAP-WT (TRAnsport in FRActured Porous media with water table boundary conditions) (Huyakorn *et al.*, 1994). This is a two-dimensional finite element model designed to simulate (non-leaky) confined and unconfined groundwater flow and single or multi-component solute transport in fractured and/or porous aquifers. Fractured porous media are represented by either the discrete-fracture or dual porosity approach, or a combination of both. When a dual-porosity mathematical idealization of a fractured medium is used, a combined two dimensional and one dimensional Galerkin finite element technique is employed to solve the governing equations for flow in the fracture and porous block domains.

The complete description of single phase fluid flow in a fractured porous medium requires two governing equations. In the discrete-fracture model, one dimension equation describes the fluid flow in the fracture and the other describes the fluid flow in the blocks. The dual-porosity model on the other hand employs two continuum equations, one for the fractures and the other for the blocks. These two equations are coupled together by a transfer function that describes mass transfer between the blocks and the fractures.

Assuming that one is dealing with a single confined fractured reservoir, the vertically integrated equation for flow in the fracture domain can be written as:

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) = S\frac{\partial h}{\partial t} - \Lambda - q \qquad \forall i = 1,2 \qquad (1)$$

where h is the hydraulic head in the fracture, T_{ij} and S are the transmissivity tensor and the storage coefficient of the fractured formation, respectively, Λ is the volumetric rate of fluid transfer per unit area, from the porous matrix blocks to the fractures, and q is the volumetric rate of fluid flow per unit area via sources such as injection wells and precipitation recharge (or sinks such as pumping wells).

According to the standard definitions of transmissivity and storage coefficient, T_{ij} and S for the fractured system can be expressed as:

$$T_{ij} = \phi_f HK_{ij} \qquad (2)$$

and

$$S = \phi HS_s \qquad \text{for a confined aquifer} \qquad (3)$$

$$S = \phi_f HS_s + S_y \qquad \text{for an unconfined aquifer} \qquad (4)$$

where H is the modelled formation, K_{ij} and S_s are the hydraulic conductivity tensorial component ij and the specific storage coefficient of representative fractures.

respectively, S_y is the specific yield of the fractured system, and ϕ_f is the fracture (or secondary) porosity of the formation. Note that for the unconfined aquifer, H corresponds to the hydraulic head above the base of the aquifer.

The term Λ in equation (1) represents the interaction between the porous rock matrix and the fractures. As the hydraulic head is reduced in the fractures, the fluid flows from the matrix to the fractures. The flux Λ is a function of the head in the fractures and the head in the porous rock matrix. Assuming that the permeability of the matrix blocks is small compared with the fracture permeability, the hydraulic head distribution in the matrix blocks can be determined by solving a simple one-dimensional transient flow problem.

For the slab block idealization, this involves the solution of:

$$\frac{\partial}{\partial \chi}\left(K'\frac{\partial h'}{\partial \chi}\right) = S_s'\frac{\partial h'}{\partial t} \tag{5}$$

For the entire saturated thickness of the formulation, H, the total rate of fluid transfer from the porous blocks to the fractures per unit area in the x_1–x_2 plane, is given by:

$$\Lambda = \left[\frac{Hb'\sigma}{b^* + b'}\right] V_n \tag{6}$$

where $V_n\sigma$ is the volumetric rate of fluid entering the fracture per unit volume of the rock matrix, b' is the characteristic length of the matrix block and b^* is the fracture aperture.

DESCRIPTION OF THE STUDY AREA

The karst aquifer of Itxina is located at the province of Vizcaya (northern Spain), in the municipality of Orozco (Fig. 1). The coordinates are N 43°03′–43°05′ and E 0°52′–0°54′. The highest elevation is Lekanda Peak (1302 m) and the lowest one is the depression of Uburun. The 6.15 km^2 and 1100 m average height aquifer is surrounded by vertical walls. Pastures and beech tree forests form an abundant vegetation. At the lowest point (725 m), to the north side of the aquifer, the Aldabide spring is the main surge and main discharge to the Itxina aquifer, with an annual mean of 250 l s^{-1} approximately over the reference period of time (1982–1983), although it has notorious changes between a few litres per second in the dry period and several m^3 s^{-1} in wet periods.

The hydraulic scheme of the karst aquifer of Itxina is a water bearing, permeable by fracturation and the karstification of the calcareous formation that, in this case, is very intensive. The operation is generally free on the side of wide conduits joining the discharge zones with the surges, which gives to the water bearing an important response capacity in the presence of recharge, as is evident through the strong oscillations in the volume of water of the spring of Aldabide, as well as a low capacity to regulate the system, as shown by the scarce volume of water collected in dry periods.

With a total runoff of 1250 mm almost all of infiltration, the minimum annual volume of inlets is 7.7 Hm^3. All the discharges from the karst aquifer of Itxina are

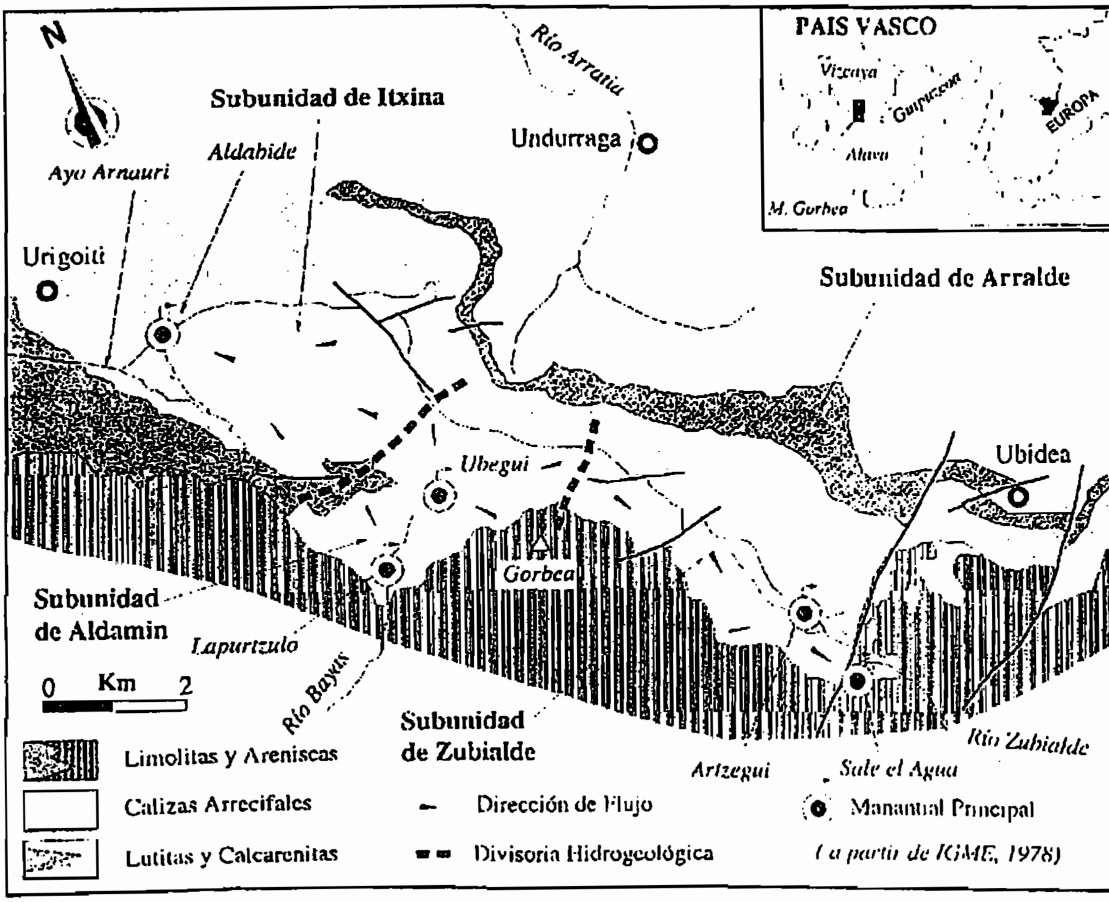

Fig. 1 Location and geological description of the study area.

produced through the springs in contact with the edge and basically 85% (5 Hm^3) of the inflows of the sector are produced through the spring of Aldabide, as can be deduced by extrapolation of the hydrological balance estimated for the annual period from October 1982 to November 1983 with the rainfalls at Gorbea station (Zastegui Valley) and the volume of water measured at Aldabide station (both stations are controlled by the Grupo Espeológico Vizcaino, GEV).

Two features should be emphasised. On one hand, the exuberance and wealth of the exokarstic formations due to the highly favourable circumstances over the Quaternary period, and on the other hand, the great development reached in a very short time by the subsurface cavities which mainly belong to the same phreatic layer suggested by Fig. 2.

The fracture network, considers that discontinuities are represented as a mixed network of fissures and conduits, also includes important vertical cavities interconnected with the horizontal galleries. It is not often taken into consideration that the circulation in the phreatic layer of a karst aquifer bearing causes the enlargement of all the rock's fissures by dissolution, which occurs at different levels in all the fissures due to the heterogeneity and anisotropy of the main factors: lithology, structure, tension conditions and the distribution of the hydraulic head potential.

In the central area of the karst aquifer all the big phreatic caves meet and it seems to have belonged to two previous layers coming from different feeding zones:

one to the south, by the Gorbea karst aquifer and the other one to the east by the Campa de Arraba (Fig. 2). They will have become shorter due to deepening of the valleys surrounding Ixtina during the Quaternary. Hence, the origin of the Itxina karst underground is likely to be related to the karst and caves of Gorbea. It may be the same age as the Mairuelegorreta Cave, an important 10 km cave in Gorbea. The clue to solve this might be found in the Zastegui Valley, which has probably disrupted the karst of both aquifers, Gorbea and Itxina, leaving them disconnected, as they appear today.

Endokarstic formations correspond to cavities which appeared in the shallow or phreatic parts, thus leaving very little space for recent surges. Examples of such formations are the phreatic born caves, made of several levels from 1150 m for the highest down to the aquifer lowest point at 725 m.

On the other side, there are some cavities at the high zone that are basically composed by pieces of phreatic galleries, shaped like exokarstic formations, in which it is common to find large tracks of flows, canals and domes, kettles and elliptical or circular sections in their galleries. The cavities located at this height on the northern slope of Itxina are old surges in which water flow has ceased due to its diminution to the base level, at the same time that the water flow has adapted to the land structure. In the same way, there are cavities at the medium zone that is composed of two big complexes, Lexardi and Gran Grieta Central.

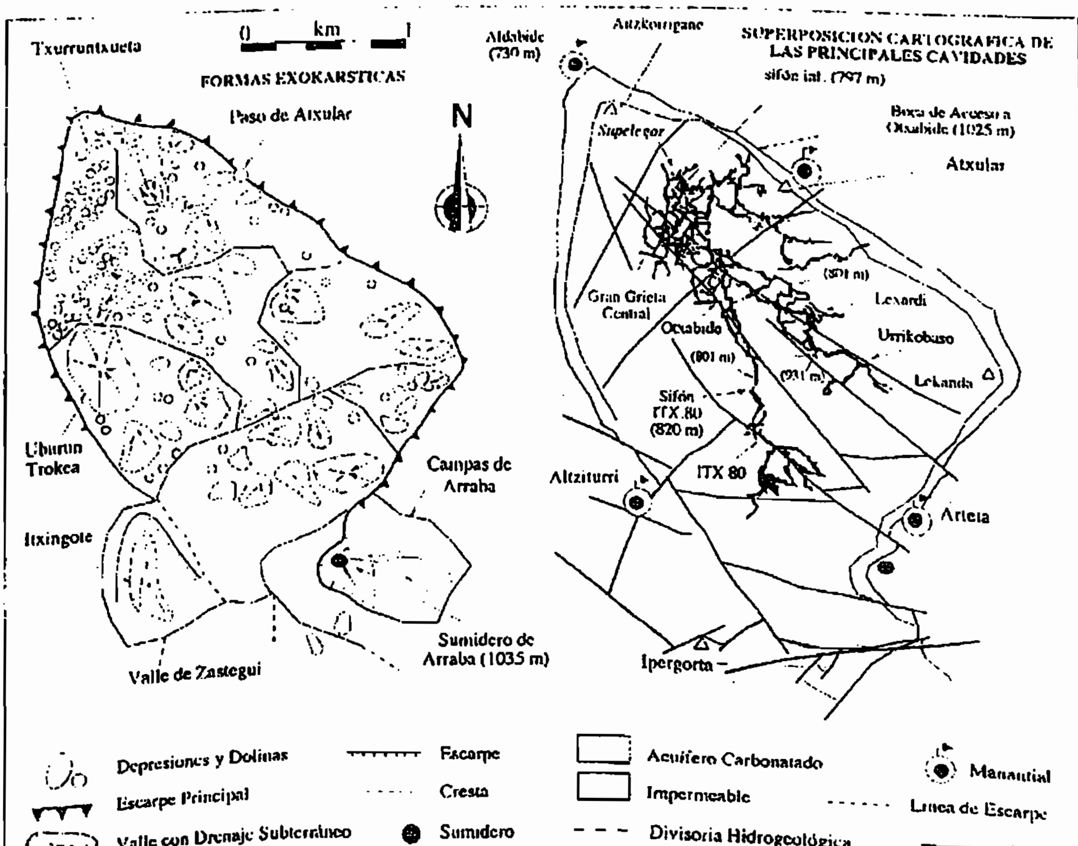

Fig. 2 Simplified plans of the morphology of the karst aquifer.

Those cavities are galleries of large dimensions reaching 30 m high for Lexardi and 40 m for Gran Grieta Central, due to the enlargement for clastic phenomenon.

Finally, there are the cavities at the deep zone including the cave of Oxtabide that has more than 6 km of galleries, with an exclusively phreatic type. At present, the main underground river that drains Itxina flows along one of those galleries, sinking in the Campa de Arraba and arising at Aldabide station.

RESULTS

To start the application of the flow model, conceptualization of the water bearing of Itxina was necessary as shown in Fig. 3. Under this scheme, the simulation of the flow system has been considered as an unconfined water bearing in stationary conditions, regarding the environment formed of parallel fractures and a slab blocks. The values of the physical parameters used in the simulations are given in Table 1. The opening of the fractures and the thickness of each prismatic block is assumed as 0.03 m and 3.05 m, respectively.

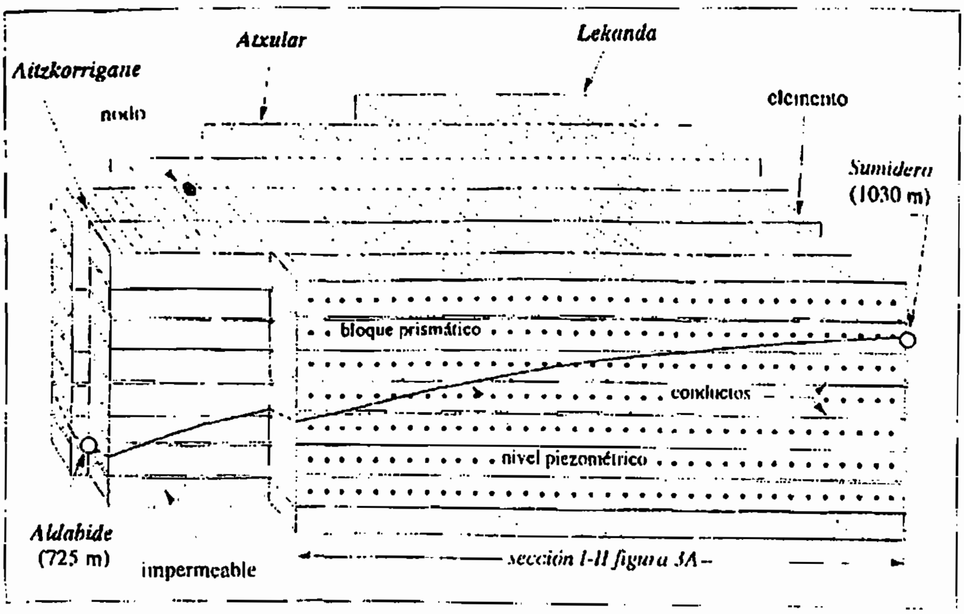

Fig. 3 Conceptualized model of the karst aquifer of Itxina.

In order to obtain a solution based on the finite element scheme, a rectangular mesh composed of 217 nodes and 180 elements has been used to represent the domain of flow in two dimensions at the surface (Fig. 3). The space between the rows and columns has been taken as constant, resulting in a mesh of seven horizontal lines and 31 vertical lines. The prismatic block was discretized to try to reproduce the form of the water bearing of Itxina, which characteristics include steep fronts in the borders that meet in the spring of Aldabide.

The lowest level of Itxina aquifer has been chosen as the reference axis, being the zero point for the system. The lowest point of the aquifer (Aldabide) is at 725 m

Table 1 Parameter values for the karstic aquifer.

Parameters of the matrix block:	
Hydraulic conductivity K_{u}	0.0001524 m day^{-1}
Hydraulic conductivity K_{v}	0.0001524 m day^{-1}
Storage coefficient S_{r}	0.0001524 m^{-1}
Length of matrix blocks b'	3.048 m
Parameters of the fractures:	
Fracture porosity ϕ_{f}	0.05
Fracture aperture b^{*}	0.006096 m
Storage coefficient S_{s}	0.012192 m^{-1}
Specific yield S_{y}	0.025
Hydraulic conductivity K_{u}	152.4 m day^{-1}
Hydraulic conductivity K_{v}	152.4 m day^{-1}

elevation and the highest point (Campa de Arraba) is at 1033 m and therefore the average aquifer height has been set at 300 m (Fig. 3).

With this information, several simulations were carried out to determine the likely distribution of the water level along the karst system. Figure 4 shows the distribution of the water levels, where the following aspects are evident: since there are no observation at the water level at the water bearing of Itxina, the only one reference point for the observation was the dynamic level of 797 m reported in the terminal siphon at the Oxtabide Cave (GEV, 1985). While observing the development of the different water levels, it is a small but important discrepancy

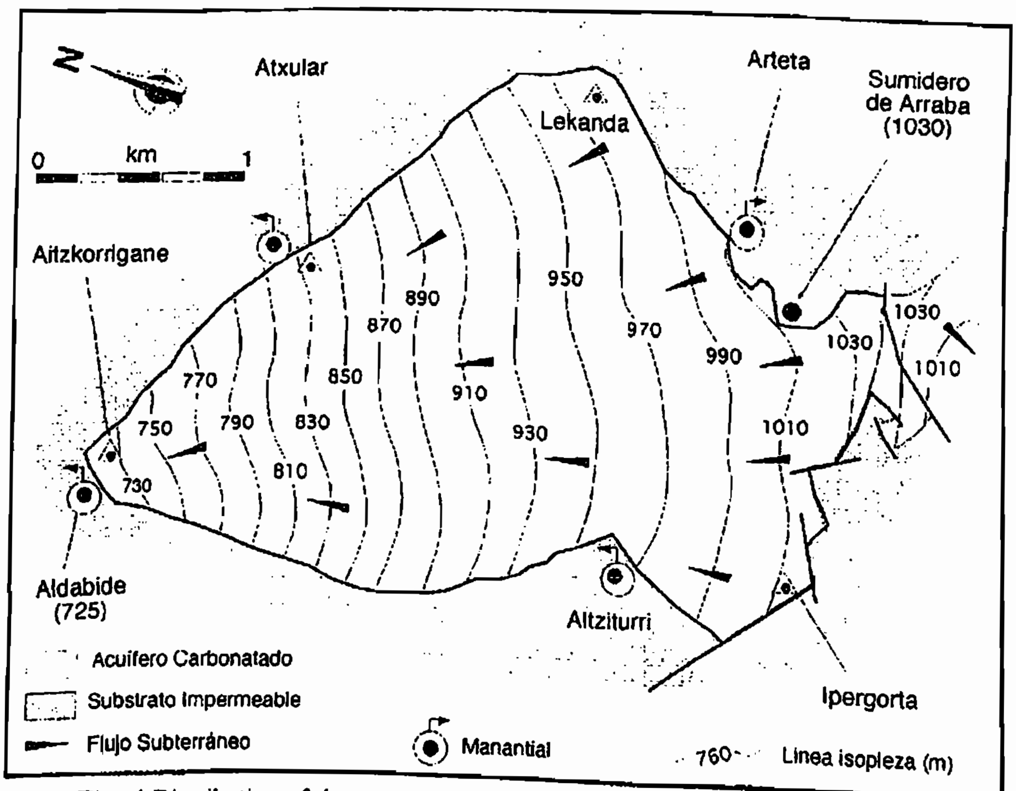

Fig. 4 Distribution of the water levels of the karst system.

arises in the position of the siphon's dynamic level (Fig. 2) and at the level estimated by the model (Fig. 4). These differences are undoubtedly explained with the variability of the environment, where the flow is associated with the distribution of the fractured which variability is highly heterogeneous.

The information available refers to the form and development of the caves (GEV, 1985) without regard to the reference levels. This represents a great limitation, since it is difficult to determine exactly the geometry of a natural fractured network. Therefore, the uncertainty in the network description introduces uncertainty to predictions by the model of the flow system. There is no doubt that the model introduces simplifications since it considers regular prismatic blocks despite the irregular form and variation of the fractures and blocks.

Under these circumstances, the flow at the spring of Aldabide was also taken into account, and once the model's stability was reached the average volume of water was 238 l s^{-1} in stationary conditions, according to the border conditions previously described. Other researchers report an average volume of water of 250 l s^{-1}, which is a different value from the one obtained from the modelling; however, the latter is related to and is between the approach limits and, therefore, it is emitter element of verification of the applied model.

Previous conditions were applied without any impulse to the system (pumping, recharge, etc.), then, the effect of the recharge was applied over the conceptualized system. Under these circumstances, a constant value of recharge was applied of 0.003048 m day^{-1}, given the unchangeable border conditions (Dirichlet condition). In these conditions, the results show a light increase in the phreatic level that causes undoubtedly an increase in the volume of water at Aldabide from 239 to 246 l s^{-1}. In spite of the addition of the recharge, the change in the phreatic level is not important, which confirms the existing lack of regulation in the system, due to the high level of fracturing.

CONCLUSIONS

A schematic representation of karst aquifers may be to consider a high hydraulic conductivity channel network with kilometre-wide intervals, surrounded by a low hydraulic conductivity fractured limestone volume and connected to a local discharge area, the karst spring. The behaviour of the karst spring (hydrographs) represents the global response of the karst aquifer to input events. However, modelling saturated flow in karstic aquifers is straightforward with few conceptual or numerical problems. At the present time, conceptual issues and/or problems in obtaining data on parameter values limit the reliability and therefore the applicability of flow models involving karst systems.

In the modelling of the flow of the karst water bearing of Itxina, the techniques employed consist of the exploration of the flow mechanism in a fractured medium. The experience of the oil industry suggests that in many cases more sophisticated formulations are needed (double porosity), although, very little research has been carried out with a natural system. In this sense, the effort involved in this research tries to conciliate the tendencies previously described in some way; however, the information problem is crucial as well as the difficulty in the measurement and estimation of the different parameters.

The system studied here is complex and very complicated to characterize, especially because of the high level of fracturing that has been obtained. Despite this, the available information has permitted application of a dual-porosity model as a result of this spring of Aldabide to the Itxina aquifer. The different simulations and variations of the impulses have permitted characterization of the system as one highly fractured with a quick response to recharge. However, the predictions under these conditions need to be considered in a situation of possible limitations, due to the fact of the estimation of the parameters, which values have been determined by comparison with other applications. These elements constitute the deficiencies of the models available today for experimental situations in laboratory and field. For these reasons, the modelling of fractured systems requires sustained effort and deep analysis to make possible its understanding.

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