PROCEEDINGS OF THE XIII INTERNATIONAL CONFERENCE ON COMPUTATIONAL METHODS IN WATER RESOURCES / CALGARY / ALBERTA / CANADA / 25 – 29 JUNE 2000

Computational Methods in Water Resources

Edited by

L.R. Bentley
University of Calgary, Alb., Canada

J.F. Sykes
University of Waterloo, Ont., Canada

C.A. Brebbia
Wessex Institute of Technology, UK

W.G. Gray
University of Notre Dame, South Bend, Ind., USA

G.F. Pinder
University of Vermont, Burlington, Vt., USA

OFFPRINT

A.A. BALKEMA / ROTTERDAM / BROOKFIELD / 2000

Computational Methods in Water Resources XIII, Bentley et al. (eds) © 2000 Balkema, Rotterdam, ISBN 90 5809 123 6

Collocation from a broad perspective

I. Herrera
Institute of Applied Mathematics and Systems (IIMAS), UNAM, México City, Mexico

ABSTRACT: Up to now, collocation has been applied by means of splines. However, a broader methodology is obtained when collocation is applied by means of fully discontinuous functions and using a domain-decomposition (or Trefftz) point of view. Then four classes of collocation methods are obtained, depending on whether collocation is directly applied to construct the solution, or it is, indirectly, applied to construct specialized test functions, by means of a Trefftz-Herrera approach, and the domain decomposition that is introduced is an overlapping or a non-overlapping one. Some of the main features of this very general methodologies are here presented.

1 INTRODUCTION

Collocation is known as an efficient and highly accurate numerical solution procedure for partial differential equations. Another attractive feature is that its formulation is very simple.

Usually this kind of methods are applied using splines. However, when formulating collocation methods, it is quite advantageous to use fully discontinuous functions -i.e., spaces in which the functions and their derivatives may have jump discontinuities- and adopt a domain-decomposition point of view. In this manner a more general method is obtained and greater versatility is achieved.

The theoretical framework for the application of fully discontinuous functions has been developed by Herrera (1985a, b, 1984, 1986, 1992, Herrera et al. 1985). Also, a concept that in some respects is broader than that of domain-decomposition method, is the concept of Trefftz method, when it is defined in a manner recently proposed (Herrera, in press). From this more general perspective the standard formulation using splines is seen as a particular case, which can be obtained when a suitable strategy for solving the final system of equations is followed.

The great generality of this framework permits classifying collocation methods into two large groups: direct and indirect methods. Direct methods are those in which collocation is used to construct the solution directly, while indirect methods correspond to Trefftz-Herrera collocation (or TH-collocation), in which collocation is used to construct specialized test functions. In particular, the conventional collocation method (Russell & Shampine 1972, de Boor & Swartz 1973, Carey & Finlayson 1975, Celia & Pinder. 1990a, b, c) is a direct method. TH-collocation (Herrera & Díaz 1999) is based on the observation that when the method of weighted residuals is applied –and this is the case of the Finite Element Method-, the information about the exact solution contained in an approximate one, is determined by the system of weighting function used. A very efficient strategy, which permits reducing the number of degrees of freedom of the global system of algebraic equations to be solved, is to apply systems of specialized test functions with the property of concentrating all the information in some of the boundaries of the subregions of the domain decomposition –they may include the internal boundaries of the finite elements and the outer boundary of the region-. Herrera's algebraic theory of boundary value problems (Herrera 1985a, b, 1984, 1986, 1992, Herrera et al. 1985) is then used as a very convenient frame-work to guide effectively the construction of such weighting functions. Each one of these methods can be applied using disjoint regions (non-overlapping) or overlapping ones. Thus, four classes of collocation methods are obtained in this manner.

When collocation methods are seen from this perspective, it becomes apparent that its study thus far has been quite incomplete, in spite of its obvious interest. Attention has been given to direct methods mainly and they have been applied using splines (Russell & Shampine 1972, de Boor & Swartz 1973, Carey & Finlayson 1975, Celia & Pinder 1990a, b,

c) which, as has already been mentioned, is quite restrictive. Due to this fact, Herrera and his collaborators have started a line of research to explore this wide class of methods and compare their relative merits. The results thus far obtained have been (Herrera, in press, Herrera & Díaz 1999) and will be reported in a series of articles that are now being prepared (Herrera, in prep., Herrera & Berlanga, in prep.). In the case of elliptic equations of second order, it is standard requiring continuity of both, the function and its derivative (Russell & Shampine 1972, de Boor & Swartz 1973, Carey & Finlayson 1975, Celia & Pinder 1990a, b, c), when formulating direct methods. However, in previous publications, (Herrera & Díaz 1999, Herrera 1995, 1997) it has been shown that these conditions can be relaxed when indirect methods are applied. Even more, in the realm of direct collocation methods it is also possible to relax such conditions when a wider class of direct methods is considered, as it is explained in this paper (see also (Herrera, in prep.)). The same is true for higher order equations and systems of equations.

The present paper is devoted to explain briefly collocation methods from this broader perspective. The theory here sketched has a very wide applicability, since it includes any partial differential equation which is linear, or systems of such equations, independently of its type. It also includes the treatment of discontinuous coefficients. In particular, it generalizes many ideas of Herrera's LAM (Localized Adjoint Method).

2 BOUNDARY VALUE PROBLEM WITH PRESCRIBED JUMPS

Consider a region Ω (Fig. 1), which may be a space-time one, and a partition $\Pi=\{\Omega_1,...,\Omega_E\}$, together with a linear space D, of functions defined in Ω. Let Σ be the internal boundary associated to the partition Π, then it is assumed that functions belonging to D, generally are discontinuous across Σ. The general problem considered in the theory of collocation methods here presented, is one with prescribed jumps, across Σ and it is defined by

$$\mathcal{L}u^i = f_\Omega^i \equiv \mathcal{L}u_\Omega^i; \quad \text{in } \Omega_i, \quad i=1,...,E \qquad (2.1a)$$

$$\mathcal{B}(u,\bullet) = g_\partial(\bullet) \equiv \mathcal{B}(u_\partial,\bullet); \quad \text{in } \partial\Omega \qquad (2.1b)$$

and

$$\mathcal{J}(u,\bullet) = j_\Sigma(\bullet) \equiv \mathcal{J}(u_\Sigma,\bullet); \quad \text{in } \Sigma \qquad (2.1c)$$

where $f_\Omega{}^i$ are given functions, while $g_\partial(\bullet)$ and $j_\Sigma(\bullet)$, are given linear functionals. These functions and functionals contain the data of the problem. In what follows, this problem will be referred as the 'boundary value problem with prescribed jumps' (BVPJ), and the symbols u_Ω, u_∂ and u_Σ will represent functions belonging to D and fulfilling Equations (2.1), which are assumed to exist. For simplicity, it will also be assumed that the BVPJ posses a unique solution fulfilling Equations (2.1) and the notation $u\in D$ will be reserved for it. Cases in which the uniqueness condition is violated, can also be treated, but the presentation of the theory would be more involved (see (Herrera 1984)).

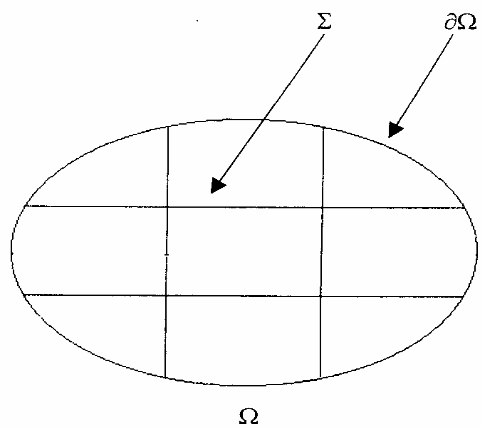

Figure 1. The region Ω.

As an illustration, consider the general elliptic equation of second order. It will be assumed that the coefficients of the differential operator may have jump discontinuities across the internal boundary Σ. Then, the boundary value problem with prescribed jumps to be considered is:

$$\mathcal{L}u^i \equiv -\nabla\bullet(\mathbf{a}\bullet\nabla u^i) + \nabla\bullet(\mathbf{b}u^i) + \mathbf{c}u^i = f_\Omega^i, \quad \text{in } \Omega_i, \quad i=1,...,E \qquad ((2.2a)$$

subjected to Dirichlet boundary conditions

$$u = u_\partial, \text{ on } \partial\Omega \qquad (2.2b)$$

and jump conditions

$$[u]=[u_\Sigma] \text{ and } [\mathbf{a}_n\bullet\nabla u]=[\mathbf{a}_n\bullet\nabla u_\Sigma], \text{ on } \Sigma \qquad (2.2c)$$

Here, as in what follows $\mathbf{a}_n \equiv \mathbf{a} \bullet \underline{n}$ and $\mathbf{b}_n \equiv \mathbf{b} \bullet \underline{n}$. When the coefficients of the differential operator are continuous, it may be seen that the conditions of Equations.(2.2c), are equivalent to prescribing the jump of the function and its normal derivative. Define the bilinear functions

$$\mathcal{B}(u,w) \equiv u(\mathbf{a}_n \bullet \nabla w + \mathbf{b}_n w) \text{ and}$$

$$\mathcal{J}(u,w) \equiv \dot{w}[\mathrm{a}_n \bullet \nabla u] - [u](\overline{\mathrm{a}_n \bullet \nabla w} + \mathrm{b}_n w), \tag{2.3}$$

and let the linear functionals $g(\bullet)$ and $j(\bullet)$ be defined by $g(\bullet) \equiv \mathcal{B}(u_\partial, \bullet)$ and $j(\bullet) \equiv \mathcal{J}(u_\Sigma, \bullet)$. Then, the BVPJ of Equations (2.2) take the form given by Equations (2.1).

3 VARIATIONAL FORMULATIONS

Introduce the following notation:

$$\langle Pu,w\rangle = \int_\Omega w\mathcal{L}u dx; \qquad \langle Q^*u,w\rangle = \int_\Omega u\mathcal{L}^* w dx, \tag{3.1a}$$

$$\langle Bu,w\rangle = \int_{\partial\Omega} \mathcal{B}(u,w)dx; \qquad \langle C^*u,w\rangle = \int_{\partial\Omega} \mathcal{C}^*(u,w)dx, \tag{3.1b}$$

$$\langle Ju,w\rangle = \int_\Sigma \mathcal{J}(u,w)dx; \qquad \langle K^*u,w\rangle = \int_\Sigma \mathcal{K}^*(u,w)dx, \tag{3.1c}$$

where $\mathcal{L}^*$ is the formal adjoint of $\mathcal{L}$, and $\mathcal{C}(w,u)$ and $\mathcal{K}(w,u)$ are suitable bilinear functions such that the real-valued bilinear functionals: P, B, J, Q*, C* and K*, fulfill the Green-Herrera formula

$$P - B - J \equiv Q^* - C^* - K^* \tag{3.2}$$

When this is the case, the BVPJ admits two variational formulations: one in terms of the data of the problem

$$< (P - B - J)^* u, w >=< f - g - j; w >; \cdots \forall w \in D_2 \tag{3.3a}$$

and the other one in terms of the sought information

$$< (Q - C - K)^* u, w >=< f - g - j; w >; \cdots \forall w \in D_2 \tag{3.3b}$$

Here, f, g and j, are linear functionals defined by

$$f \equiv Pu_\Omega, \quad g \equiv Bu_\partial, \quad j \equiv Ju_\Sigma \tag{3.4}$$

These two variational formulations of the BVPJ are equivalent, by virtue of the identity of Equation (3.2). Also, such variational principles are applicable even if the coefficients of the differential operators are discontinuous.

In the case of the general elliptic equation of second order, in which the differential operator $\mathcal{L}$ is given by Equations (2.1), one has

$$\mathcal{L}^* w \equiv -\nabla \bullet (\underline{\mathbf{a}} \bullet \nabla w) - \underline{\mathbf{b}} \bullet \nabla w + \mathbf{c}w, \tag{3.5}$$

for the formal adjoint and Equation (3.2) is a Green-Herrera formula when

$$\mathcal{C}(w,u) \equiv w\mathbf{a}_n \bullet \nabla u \text{ and}$$

$$\mathcal{K}(w,u) \equiv \dot{u}[\underline{\mathrm{a}}_n \bullet \nabla u] - [w](\overline{\underline{\mathrm{a}}_n \bullet \nabla u - \mathrm{b}_n u}), \tag{3.6}$$

In particular, when $\underline{\mathbf{b}} \equiv 0$, the differential operator $\mathcal{L}$ is formally symmetric, B≡C and J≡K. Even more, for functions which vanish on the external $\partial\Omega$ and are continuous across Σ, *P-B-J≡Q-C-K* is positive definite and a maximum principle is applicable.

4 CLASSIFICATION OF COLLOCATION METHODS

The method proposed originally by Trefftz, (1926), has been generalized very much and the following definition, that was introduced recently by the author (Herrera, in press, Herrera, in prep.) to apply in this more general context, is quite suitable for our purposes.

Definition 4.1.- Let $\Pi=\{\Omega_1,...,\Omega_E\}$ be a partition of Ω and for every i=1,...,E, let $\mathcal{H}_i$ be defined by the condition that $u^i_H \in \mathcal{H}_i$, if and only if, $\mathcal{L}u^i_H = 0$, in Ω_i. In addition, let $\mathcal{H} \equiv \mathcal{H}_1 \oplus ... \oplus \mathcal{H}_E$. Then the problem of finding, such that

$$u = \sum_{i=1}^E u^i_\Omega + \sum_{i=1}^E u^i_H = u_\Omega + u_H; \tag{4.1}$$

is solution of the Boundary Value Problem with Prescribed Jumps (BVPJ), will be referred as "Trefftz Problem".

Generally what is required is to obtain $u_H \equiv \Sigma_{i=1}^E u^i_H$, since $u_\Omega \equiv \Sigma_{i=1}^E u^i_\Omega$, is a datum of the problem, and the notation u_H will be reserved for the solution of Trefftz problem. A full treatment of

Trefftz problem requires the introduction of the concept of Trefftz-Herrera (TH)-completeness, for families of functions fulfilling $\mathcal{L}u^i_H = 0$ (Herrera 1980, Begehr & Gilbert 1992).

The general classification of collocation procedures into direct and indirect methods, derive from the corresponding approaches to Trefftz problem, which are the direct (or Trefftz-Jirousek) and indirect methods (or Trefftz-Herrera). The essential feature of direct methods is that the linear combination which fulfills Equation (4.1), is determined by imposing directly the boundary and jump conditions of Equations (2.7a, b). In Trefftz-Herrera method, on the other hand, special test or weighting functions are applied to obtain enough information on the internal boundary Σ, so as to define well posed problems in each one of the subregions Ω_i, $i = 1, \ldots, E$.

When Trefftz method is combined with the method of collocation, two very broad classes of numerical methods are obtained, and this paper is devoted to discuss them. Most of the work done with direct Trefftz methods, so far, has been based on the application of analytical solutions (Jirousek & Wróblewski 1996), but Herrera and his collaborators are working in a general class of collocation methods based in this approach (Herrera & Herrera, in prep.). These will be referred as direct collocation methods. The use of collocation in combination with Trefftz-Herrera method, was first presented in (Herera 1987) and will be referred as Trefftz-Herrera (TH)-collocation. Of course, it is also possible to use analytical solutions when applying Trefftz-Herrera method, as has been done in Celia and Herrera. (1987), but the versatility of collocation is, by far, superior. A similar comment applies in the case of Jirousek-Trefftz method, although numerical corroboration is still lacking.

5 DIRECT METHODS

The application of direct methods to one dimensional problems is relatively straight-forward (Herrera & Herrera, in prep.). However, their application in several dimension is considerably more complicated and requires of the use variational principles (Jirousek & Wróblewski 1996). They can be specific for the particular differential equation considered, as those presented in Section 3. An approach possessing also great generality, but with the additional advantage of yielding symmetric and positive definite matrices, is the use of least-squares (Herrera & Herrera, in prep.).

In general non-overlapping procedures require solving greater systems of equations. Overlapping methods, on the other hand, permit reducing the number of degrees of freedom because some of the jump conditions, such as continuity conditions, can be fulfilled from the start. This will be discussed further in Section 7.

Consider, as an example, the BVPJ for the general elliptic equation of second order, defined by Equations (2.2). When a direct collocation method is applied, one can use the variational principle in terms of the data of the problem of Equation (3.3a), with help of Equation (2.3). Other possibility is to apply least squares to the quantities $[\hat{u}-u_\partial]$, on $\partial\Omega$, together with $[\hat{u}-u_\Sigma]$ and $[\underline{a}_n\bullet\nabla\hat{u}-\underline{a}_n\bullet\nabla u_\Sigma]$, on Σ, where $\hat{u}\in D$ is any trial function. When the coefficients of the differential operator are continuous, it is simpler, to replace this latter quantity by $[\partial\hat{u}/\partial n-\partial u_\Sigma/\partial n]$. In addition, the following observation must be made: when the numerical method that is applied to solve the local problems is collocation, the boundary condition $\hat{u}=u_\partial$, on $\partial\Omega$, can be fulfilled by the trial functions from the start, so that the least squares on $[\hat{u}-u_\partial]$, need not be applied. Finally, when overlapping methods are used, it is easy to construct trial functions which fulfill the condition $[\hat{u}-u_\Sigma]$, on Σ (see, Section 7), and this reduces the number of degrees of freedom of the matrices of the global system of equations.

It must be mentioned that an alternative procedure, although in some sense, this is only semi-direct, is an overlapping procedure in which the local solutions are used to impose compatibility conditions, from which the global system of algebraic equations is derived (Herrera, in prep.). It may be shown that this is the basic method which is behind the well-known Schwarz alternating procedure (Lions 1987).

As an illustration, of this semi-direct procedure, consider the equation $\mathcal{L}u = 0$ in an interval of the real line, where $\mathcal{L}$ is a second order operator. Let $x_i\in(x_{i-1}, x_{i+1})$, then $u(x_i)$ depends linearly on $u(x_{i-1})$ and $u(x_{i+1})$. Indeed, $u(x_i) = \varphi^-_i(x_i)u(x_{i-1})+\varphi^+_i(x_i)u(x_{i+1})$, and this equation constitutes a three-diagonal system of equations, which coefficients can be obtained solving locally, by collocation, a pair of boundary value problems in (x_{i-1}, x_{i+1}): $\mathcal{L}\varphi^-_i = \mathcal{L}\varphi^+_i = 0$, subjected to $\varphi^-_i(x_{i-1})=\varphi^+_i(x_{i+1})=1$ and $\varphi^-_i(x_{i+1})=\varphi^+_i(x_{i-1}) = 0$. The generalization of this procedure will be presented in (Herrera, in prep.).

6 TH-COLLOCATION: INDIRECT COLLOCATION

With reference to the variational formulation in terms of the sought information Equation (3.3a), the term C*u contains the complementary boundary values (Herrera 1985a), in the external boundary $\partial\Omega$, while the information in the internal boundary Σ, is contained in K*u. In applications to elliptic problems, usually one is interested in obtaining information on Σ, exclusively, since the full solution can be reconstructed by solving local problems. The solution of such local problems constitute an optimal interpolation (Herrera 1993).

However, in time dependent problems the final state of the system under study, which is part of the external boundary of the space-time region, is frequently of special interest. Such information is contained in C*u. Thus, in general, one is interested not only in K*u, but also in C*u.

Due to these facts, a general and systematic formulation of Trefftz-Herrera collocation, requires the introduction of a decomposition of the bilinear functional C+K, of the form

$$C + K \equiv S + R \qquad (6.1)$$

such that S^*u contains precisley the information that is sought, which as has already been said, may contain information in $\partial\Omega$, Σ, or in both. Any function $\tilde{u} \in D$, such that

$$S^*\tilde{u} = S^*u; \qquad (6.2)$$

is said to 'contain the sought information' and in what follows, the symbol $\tilde{u}$ is reserved for functions with this property.

Usually, especially in elliptic problems, S is taken so that the sought information, S*u, defines well-posed problems in each one of the subregions of the partition, since this allows introducing an optimal approximation in the sense mentioned above.

Let $N_Q \subset D$ and $N_R \subset D$, be the null subspaces of Q (i.e., $\mathcal{L}^*w$=0, in Ω_i, i=1,...E) and R respectively. In order to formulate a necessary and sufficient condition for a function to contain the sought information, it has been necessary (Herrera, in press, Herrera, in prep.) to formulate an extension of the concept of completeness that was introduced by the author in (Herrera 1980) and which has been very effective in the study of complete families (Begehr & Gilbert 1992). This extended concept is:

Definition 6.1.- A subset of weighting functions, $\mathcal{E} \subset N_Q \cap N_R \subset D$, is said to be TH-complete for S^*, when for any $\hat{u} \in D$, one has:

$$< S^*\hat{u}, w > = 0, \ \ \forall w \in \mathcal{E} \Rightarrow S^*\hat{u} = 0; \qquad (6.3)$$

Clearly, a necessary and sufficient condition for the existence of TH-complete systems, is that $N_Q \cap N_R$, itself, be a TH-complete system and this assumption is fulfilled in many problems of interest.

Theorem 6.1.- Let $\mathcal{E} \subset N_Q \cap N_R$ be a TH-complete system of weighting functions and assume that there exists a BVPJ solution $u \in D$. If $\hat{u} \in D$, then a necessary and sufficient condition for $\hat{u}$ to contain the sought information, is that

$$- < S^*\hat{u}, w > = < f - g - j, w >, \cdots \forall w \in \mathcal{E}; \qquad (6.4)$$

Theorem 6.1, constitutes the basis of indirect methods, because after discretization it yields the basic system of equations to be solved.

It is useful to introduce also a decomposition of K, given by

$$K \equiv K^S + K^C \qquad (6.5)$$

Using it one defines

$$S \equiv K^S \text{ and } R \equiv C + K^C, \qquad (6.6)$$

then the sought information, $S^*u \equiv K^{S*}u$, contains information in the internal boundary, exclusively. In elliptic problems the choice $K^C=0$ leads to non-overlapping indirect methods, while $K^C \neq 0$ corresponds to overlapping indirect methods. Also, when $K^C \neq 0$ the sought information is less than when $K^C=0$ and again, like in the case of direct methods, this fact yields numerical advantages for the overlapping methods, which are not exhibited by the non-overlapping ones.

As in Section 5, let us illustrate the TH-collocation by applying it to the elliptic BVPJ of second order of Equations (2.2). It was mentioned before, that in the case of elliptic problems a convenient strategy is to concentrate all the sought information on Σ. This implies using the definitions of Equation (6.6). A first possibility is to set $K^C \equiv 0$; i.e., $S \equiv K$ and the test functions are required to fulfill $\mathcal{L}^*w$=0, in each one of the subregions separately, together with w = 0, on $\partial\Omega$. Observe that no matching condition between the subregions is imposed. Thus, in this case the method is non-overlapping. The sought information is $\dot{u}$ and $\bullet/(\partial u/\partial n)$, on Σ. This information is excessive, in the sense that when it is used to define local boundary value problems, they turn out to be over-determined. Indeed, it would be enough, for example, to prescribe $\dot{u}$ on Σ, to have a well posed problem, if that information is complemented with the data, on $\partial\Omega$, of the problem (see Fig.1).

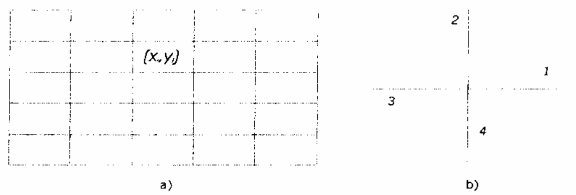

Figure 2a) Rectangular domain decomposition of Ω; b) Numbering of internal boundaries.

Thus, one strategy which permits handling information which is essential only, is to concentrate all the information in $\dot{u}$ on Σ. This is achieved if one sets

$$< K^{C} w,u >= -\int_{\Sigma}[w](\underline{a}_n \bullet \nabla \dot{u} - \underline{b}_n u)\,dx, \qquad (6.7)$$

in Equation (6.6). In this case, the requirement $w \in N_R$ implies the condition $[w]=0$, on Σ, in addition to the previous conditions. Thus, such functions must be continuous across Σ. The construction, by collocation, of test functions fulfilling these conditions is not difficult, but requires putting together several subregions. Thus, diminishing the information that has to be handled, and so the degrees of freedom, leads to an overlapping method. In Section 7, a brief discussion about the construction of the test functions is presented.

7 CONSTRUCTION OF TH-COMPLETE SYSTEMS

Consider again the BVPJ for the general elliptic equation of second order. For simplicity, a rectangular region will be considered and the subregions of the partition, will be rectangles (Fig. 2a).

What is required, for a system of functions to be TH-complete is that, for each subregion Ω_i, the traces of its members span $H^0(\partial\Omega_i)$. When collocation methods are used in the construction of TH-complete systems, one may choose a system of functions which spans $H^0(\partial\Omega_i)$ and then solve a family of boundary value problems taking as boundary conditions each one of the members of such system. A convenient choice for the system of functions that spans $H^0(\partial\Omega_i)$, is a system of piece-wise polynomials. A linear basis of such system of polynomials may be obtained taking the four bilinear polynomials which have the property of assuming the value 1 at one corner of each given quadrilateral and vanishing at all the other three corners, together with all the piecewise polynomials defined on $\partial\Omega_i$, which vanish identically at three sides of the quadrilateral.

For constructing a TH-complete system, fulfilling a continuity condition, collocation methods are also quite suitable. With each internal node (x_i,y_i) a region Ω_{ij}, which is the union of the four rectangles of the original partition that surround that node, is associated. Then, the system of subregions $\{\Omega_{ij}\}$ is overlapping. The boundary of Ω_{ij} is $\partial\Omega_{ij}$, while that part of Σ laying in the interior of Ω_{ij} will be denoted by Σ_{ij} (Fig. 2b); it is constituted by four segments which will be numbered as indicated in Figure 2b and form a cross. Given any sub-region $\partial\Omega_{ij}$, a system of functions which fulfill $\mathcal{L}^{*}w = 0$ in its interior and vanish on $\partial\Omega_{ij}$, is developed. Using the numbering already introduced, with each interior node (x_i,y_j) five groups of weighting functions are constructed, which are identified by the conditions satisfied on Σ_{ij}:

Group 0 — This group is made of only one function, which is linear in each one of the four segments of Σ_{ij} and $w_{ij}(x_i,y_j) = 1$.

For N=1,...,4, they are defined by:

Group N — The restriction to interval "N", of Figure 1b, is a polynomial in x, which vanishes at the end points of interval "N". For each degree ≥ 2, there is only one linearly independent polynomial.

The support of the test function of Group "0", is the whole square, while those weighting functions associated with Groups "1" to "4", have as support rectangles which can be obtained from each other by rotation, as it is shown in (Fig. 3).

Of course, when developing numerical algorithms for the solution of boundary value problems, only a few terms of these TH-complete systems are taken; it could be only one (see (Herrera & Díaz, in prep)). Generally, the order of precision of the resulting scheme will depend on the number of terms taken.

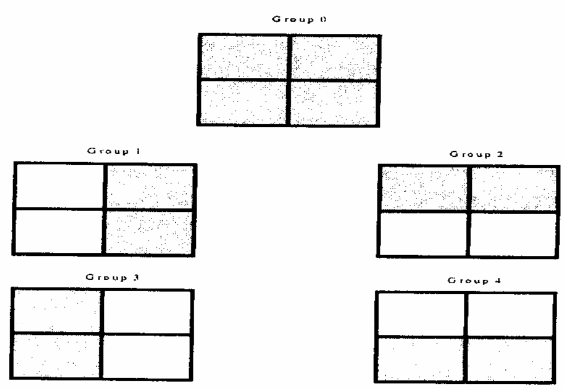

Figure 3. The five groups of weighting functions according to their supports.

8 CONCLUSIONS

The span of collocation methods is widened very much when they are formulates using fully discontinuous functions, both as base and test functions, and a Trefftz approach (or domain decomposition) is adopted. The four group of collocation methods thus far identified –direct-non-overlapping, direct-overlapping, indirect-non-overlapping and indirect overlapping-, must be fully developed and their properties investigated more deeply. Research in this direction is under way. Overlapping methods have shown to have the ability of reducing the number of degrees of freedom to be handled, and this yields important numerical advantages.

REFERENCES

Begehr, H. & R.P. Gilbert 1992. *Transformations, transmutations, and kernel functions.* Vol. 1. Longman Scientific & Technical.

Carey, G.F. & B.A. Finlayson 1975. Orthogonal collocation on finite elements. *Chem En. Sci.* 30: 587-596.

Celia, M. & I. Herrera 1987. Solution of general differential equations using the algebraic theory approach. *Journal of Numerical Methods for Partial Differential Equations.* 3(1): 117-129.

Celia, M.A. & G.F. Pinder 1990a. Generalized alternating-direction collocation methods for parabolic equations. I. Spatially varying coefficients. *Numerical Methods for Partial Differential Equations.* 6: 193-214.

Celia, M.A. & G.F. Pinder 1990b. Generalized alternating-direction collocation methods for parabolic equations. II. Transport equations with applications to seawater intrusion problems. *Numererical Methods for Partial Differential Equations.* 6: 231-243.

Celia, M.A. & G.F. Pinder 1990c. Generalized alternating-direction collocation methods for parabolic equations. III. Nonrectangular domains. *Numerical Methods for Partial Differential Equations.* 6: 231-243.

De Boor, C. & B. Swartz 1973. Collocation at Gaussian points. SIAM *Journal Numer Anal.* 10: 582-606.

Herrera, I. 1980. Boundary methods. A criterion for completeness. Proceedings of the National Academy of Sciences, USA. 77(8): 4395-4398.

Herrera, I. 1984. *Boundary methods. An algebraic theory.* Pitman Advanced Publishing Program, Boston, London, Melbourne

Herrera, I. 1985a. Unified approach to numerical methods. Part 1. Green's formulas for operators in discontinuous fields. *Journal of Numerical Methods for Partial Differential Equations.* 1(1): 12-37.

Herrera, I. 1985b. unified approach to numerical methods. Part 2. Finite elements, boundary methods and its coupling. *Journal Numerical Methods for Partial Differential Equations.* (3): 159-186.

Herrera, I.; L. Chargoy & G. Alduncin 1985. Unified approach to numerical methods. Part 3. Finite differences and ordinary differential equations. *Journal of Numerical Methods for Partial Differential Equations.* 1(4): 241-258.

Herrera, I. 1986. Some unifying concepts in applied mathematics. The merging of disciplines: new directions in pure, applied and computational mathematics. In R.E. Ewing et al. (eds), Springer Verlag, New York. :79-88.

Herrera, I. 1987. The algebraic theory approach for ordinary differential equations: Highly accurate finite differences. *Numerical Methods for Partial Differential Equations.K 3:199-218.*

Herrera, I. 1992. Localized adjoint methods: a new discretization methodology. Chapter 6 of the Book: Computational Methods in Geosciences. In W.E. Fitzgibbon & M.F. Wheeler (eds), SIAM. :66-77.

Herrera, I. 1993. Innovative discretization methodologies based on LAM. Finite Elements in Fluids: New Trends and Applications, Vol. 2. In K. Morgan et al. (eds). Pineridge Press. :1437-1447.

Herrera, I. 1995. Trefftz-Herrera domain decomposition. Special Volume on Trefftz Method: 70 Years Anniversary. *Advances in Engineering Software.* 24: 43-56.

Herrera, I. 1996. Trefftz method, domain decomposition and TH-collocatioin. XI Inter Conf Comp Meth Water Res, Cancún, México. 1:515-524.

Herrera, I., J. Hernández, A. Camacho & J. Garfias 1997. Parallelization using TH-collocation. In F. García et al. (eds). Proceedings of the 2nd UNAM-CRAY Supercomp Conf Cambridge University Press, New York. : 266-275.

Herrera, I. 1997. Trefftz-Herrera method. *Comput Assist Mech Eng Sci.* Polish Academy of Sciences. 4:369-382.

Herrera, I. & M. Díaz 1999. Indirect methods of collocatioin: Trefftz-Herrera collocation. *Numerical Methods for Partial Differential Equations.* 15(6): 709-738.

Herrera, I. On Jirousek method and its generalizations. Special issue of *Computer Assisted Mechanics and Engineering Sciences.* (in press).

Herrera, I. Trefftz method: A general theory. (in prep.).

Herrera, I. & R. Berlanga. Operator extensions (in prep.).

Herrera, I. & M. Díaz. Trefftz-Herrera collocation in two dimensions. (in prep.).

Herrera, I. & G. Herrera. Collocation: direct methods. (in prep.).

Jirousek, J. & A. Wróblewski 1996. T-elements: state of the art and future trends. *Archives of Computational Methods in Engineering.* 3,4: 323-434.

Lions, P.L. 1987. On the Schwarz alternating method. I. First International Symposium on Domain Decomposition Methods for Partial Differential Equations. In R. Glowinski et al. (eds. :1-42.

Russell, R.D. & L.G. 1972. Shampine. A collocation method for boundary value problems. *Numer Math.* 19: 1-28.

Trefftz, E. 1926. Ein Gegenstuck zum Ritz'schen Verfahren. Proceedings of the 2nd Int Cong Appl Mech. Zurich. :131-137.

FROM THE SAME PUBLISHER

Hofwegen, Paul J.M.van & Frank G.W.Jaspers 90 5410 472 4
Analytical framework for integrated water resources management – *Guidelines for assessment of institutional frameworks*
(IHE monographs 2)
1999, 25 cm, 110 pp., EUR 58.00 / $68.00 / £41
St.edn:90 5410 473 2, EUR 32.00 / $38.00 / £23
The analytical framework; Integrated water resources management - general principles; IWRM requirements for the functional levels; Interventions in IWRM capacity building; Assessment of the institutional framework - process and tools; etc.

Batuca, Dan G. & J.M.Jordaan, Jr. 90 5410 477 5
Silting and desilting of reservoirs
2000, 25 cm, c.330 pp., EUR 95.00 / $99.00 / £63
Created by river damming and aiming to store water for as long as possible till needed and used, reservoirs have been built since immemorial times for irrigation, water supply, storage, flood control, navigation and power generation, as part of mankind's strategy for survival. Silting of a reservoir is an unavoidable and irreversible process, naturally-caused and anthtopically-influenced, that can not be interrupted or stopped but can be slowed down and controlled to a certain extent by various soil conservation practices and agricultural works in the catchment, by means of some engineering structures in the river system, as well as by adequate strategies of reservoir operation. Whenever silting control is ineffective or very costly then removal of deposited sediment by hydraulic or mechanical desilting techniques is the only way to ensure a long lifespan for reservoirs. Topics covered: General considerations about reservoirs; Hydraulics of reservoirs; Sediment transport in reservoirs; Silting of reservoirs. Process, causes & effects; Sediment distributions in reservoirs Calculation and prediction; Coping & dealing with silting of reservoirs; Control of reservoir silting; Desilting of reservoirs; etc.

Thompson, Stephen A. 90 5410 435 X
Hydrology for water management
1999, 25 cm, 476 pp., EUR 80.00 / $95.00 / £56
Student edn: 90 5410 436 8, EUR 45.00/$49.00/£32
This book provides a foundation for understanding hydrologic principles, and some of the more commonly-used methods of hydrologic analysis. The approach is a blend of quantitative and qualitative discussion of applied hydrologic issues. Embedded worked examples guide the reader through the various techniques, while step-by-step spreadsheet (EXCEL) exercises develop the basic skills for using computer spreadsheets for solving hydrologic problems. The book contains over 170 line drawings and maps, while over 100 tables summarize information. Author: Millersville Univ., Pennsylvania, USA.

Alaerts, G.J., F.J.A.Hartvelt & F.-M.Patorni (eds) 90 5410 421 X
Water sector capacity building: Concepts and instruments – *Proceedings of the second UNDP symposium on water sector capacity building, Delft, 1996*
(IHE proceedings,)
1999, 25 cm, 574 pp., EUR 84.00 / $90.00 / £59
Capacity building of institutions and of individuals is a key mechanism to improve public governance. In the water sector, which encompasses water management and water services such as water supply, sanitation and irrigated agriculture, capacity building is seen as the solution to the main weaknesses in the sector: weak institutional frameworks, poorly targeted organizations with inappropriate skills mixes, and, in general, no orientation to sustainable 'demand driven' management. This volume contains the papers and the outcome of workshops of the second UNDP Water Sector Capacity Building Symposium (Delft, The Netherlands, 1996). Whereas the First Symposium focused on the nature and broad strategic outlines of the capacity building concept, this book reviews the first field experiences and discusses the tools and approaches. The contributions make a specific attempt to discuss capacity building as a means to support and guide sector and policy reform.

Nunes Correia, Francisco (ed.) 90 5410 438 4
Water resources management in Europe – *Institutions, issues and dilemmas*
1998, 25 cm, 1112 pp., 2 vols, EUR 177.00/$206.00/£125
Volume 1: This book is based on a systematic focused comparative analysis of water institutions in Europe, considering five countries as case studies: France, Germany, the Netherlands, Portugal and the United Kingdom. An in–depth analysis is presented in five country reports complemented by an overview of water availability, uses and institutions. Contents: Introductions; General framework for water planning and management; Availability and uses of water; Public and private actors and their roles at different levels of water planning and management; Main functions in water planning and management; Relevant trends in water planning and management; Global appraisal of the institutional system for water planning and management.
Volume 2: In this volume 10 topics of European and world wide relevance are presented and discussed. The book is structured around issues and not on basis of specific country reports. It is an attempt of shedding light onto the many problems that are common to all countries, although they may be faced in different places with different degrees of relevance and urgency. Contents: Water resources information policy; River basin planning and management; Transboundary issues in water resources; etc.